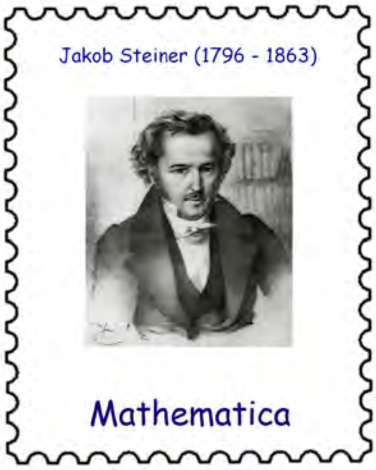
Jakob Steiner (March 18, 1796 – April 1, 1863)

by Heinz Klaus Strick, Germany



(chalk drawing by Nikolaus Senn, Burgerbibliothek Bern)

Jakob Steiner grew up as the youngest of eight children on his parents' farm in Utzenstorf (near Bern). In the first few years his lessons in a primary school were limited to memorising the cathechism and the hymn book. It was not until he was 14 that Jakob learned to read and write and he taught himself arithmetic by selling the farm's produce at the market.

At the age of 18, the inquisitive youngster left home to attend the school of the reforming teacher Johann Heinrich Pestalozzi in Yverdon in the canton of Vaud.

The teaching and learning methods practised at the school ("*learning with head, heart and hand*") helped to develop his abilities and after only one and a half years Jakob was assigned to take over teaching.

When the school in Yverdon closed, Steiner went to Heidelberg to study mathematics and he earned his living as a private teacher. In 1821 he accepted an offer to work as a substitute teacher at the *Friedrichswerderschen Gymnasium* in Berlin.

However, he did not stay there for long: the headmaster, himself a mathematician, insisted that the lessons be conducted exactly according to the textbook he himself had written. But Steiner was convinced of Pestalozzi's method of letting the pupils discover mathematical facts for themselves as much as possible, and since this could not be reconciled with the book's approach, he was dismissed after a few weeks.

The self-taught Steiner was anxious to get a formal, official licence as a teacher. He had no problems with the requirements in mathematics, but failed because of a lack of knowledge in philosophy, history and literature, which were expected of him.

He enrolled at Berlin University and he earned his living as a private teacher, including in Wilhelm von Humboldt's house, until he finally found a position as an assistant teacher at the newly established *Trade Academy* (similar to a later Senior Secondary School).

Steiner's headmaster, Karl Friedrich Von Klöden, also a mathematician and also a follower of Pestalozzi's teaching methods, gradually even let him teach the senior classes.

In 1826 August Leopold Crelle founded the Journal *Für die reine und angewandte Mathematik*. Steiner wrote four articles for the first issue of the journal and in the course of the years 58 more were added. One of these papers dealt with the question of how many "parts" space can be divided into by *n* planes. Steiner showed that the maximum number is $\frac{1}{6}\cdot(n^3+5n+6)$.

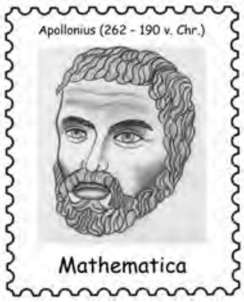
Niels Henrik Abel also published several articles in Crelle's Journal (before his early death in 1829). When Crelle was seen in public in Berlin together with the two young scholars, people remarked *Adam is once again walking with his sons Cain and Abel.*

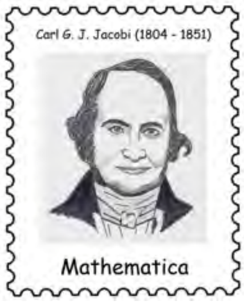
In 1826, Steiner finally received an official teaching permit and he impressed the responsible commission with the articles published by Crelle.

As he wanted to elaborate his ideas on geometry and publish them in book form, he applied to the Ministry to grant him a subsidy for the printing costs. In the application he wrote:

> *The new way of looking at things, which concerns the geometric-synthetic method itself, differs, by the way, from the older method in particular in that it does not merely prove, as the latter does, the connection of individually formed propositions, and testify to the truth of the one by that of the other: but it lets the propositions arise directly from one another, and reveals by a genetic course in a more complete way the form of synthetic arising, than the older method, common since the Greeks.*

Klöden approved the proposal with enthusiasm and he even placed Steiner's manuscripts above the lost writings of Apollonius of Perga.





(drawings © Andreas Strick)

Friedrich Wilhelm Bessel, professor in Königsberg, asked for an opinion, remarked

> *... even if he certainly knows Poncelet's work, his essays have so much originality that one must regard them as independent and admire in particular the imagination with which he knows how to visualise the properties of figures and surfaces.*

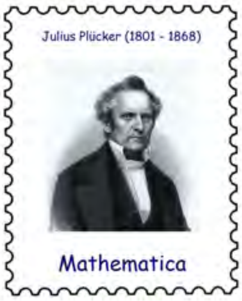
The French mathematician and engineer Jean-Victor Poncelet had laid the foundations of projective geometry in 1823 with his *Traité des propriétés projectives des figures*.

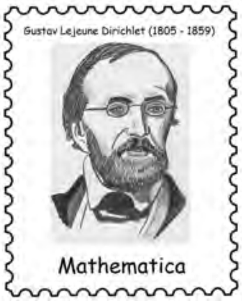
In his first book in 1832: *Systematische Entwickelung der Abhängigkeit geometrischer Gestalten von einander* (Systematic development of the dependence of geometric figures on each other) Steiner consistently developed these ideas further.

Bessel and Carl Gustav Jacob Jacobi (with whom Steiner had been friends since his student days at the University of Berlin and who had also been a professor in Königsberg since 1827) ensured that this ingenious contribution by Steiner received special recognition in the form of an honorary doctorate from the University of Königsberg.

In the meantime, however, Steiner's position at the *Trade School* – despite his promotion to senior teacher – had changed dramatically. Complaints about his rude and insulting manner in dealing with pupils and colleagues became more and more frequent. The headmaster Klöden saw his official authority undermined when Steiner changed the lesson content of his classes without consultation and even declared his newly published book to be the compulsory textbook for his senior classes. Finally, he was even threatened with dismissal from the school service.

After the publication of his second book *Die geometrischen Constructionen ausgeführt mittels der geraden Linie und eines festen Kreises* (Geometric constructions executed by means of the straight line and a fixed circle) in 1833, STEINER, overburdened by teaching and scientific work at the same time, applied for convalescent leave and returned to his old home in Switzerland. At the newly founded University of Bern he held talks about a possible professorship, partly in order to have a better negotiating position after his return to Berlin. When one of Berlin university's full professors finally died and another (JULIUS PLÜCKER) accepted a call to Halle, the longed-for path was cleared for STEINER.





ALEXANDER and WILHELM VON HUMBOLDT saw to it that an extraordinary professorship was established for him (1834). CRELLE and GUSTAV LEJEUNE DIRICHLET saw to it that STEINER was admitted to the *Prussian Academy of Sciences*.

STEINER's second major publication also caused a sensation; it dealt with the constructions of EUCLIDean geometry. He showed that a ruler (without any markings) is sufficient for these as soon as a fixed circle is drawn, so that one can dispense with the further use of a compass. In the years that followed, however, only smaller treatises appeared and the seven-volume work on geometry that he had announced in 1830 never materialised. Posthumously published elaborations of his lectures gave an idea of what he planned in detail.

STEINER did not care much for algebra and analysis. He was of the opinion that the "calculating" customary in these disciplines replaced thinking, whereas the preoccupation with the constructions and conclusions of synthetic geometry stimulated thinking – in contrast to analytical geometry, in which "calculating" also took place.

> *Analysis pulls the sleeping cap over your head. With us it's a matter of: Open your eyes, then you can see things.*

Since ancient times, geometers had been dealing with the so-called *isoperimetric* problem (*iso* = equal, *perimeter* = circumference):

- *What is the shape of a closed curve with a fixed length that encloses the largest area?*

During a stay in Paris in 1840/41, he published five proofs that the circle has this property (equivalently: of all curves that enclose the same area, the circle has the smallest circumference), including a purely geometric proof. He did this by showing that the enclosing curve must be convex and symmetrical in any direction. (KARL WEIERSTRASS closed a gap in the proof, namely showing that a solution existed at all).

STEINER also proved that among all bodies with a fixed surface the sphere had the largest volume (equivalently: ... with a fixed volume the smallest surface).

In 1848 he published a short but ingenious paper *General properties of algebraic curves* – but without proofs. Generations of later mathematicians would be busy verifying the correctness of his statements.

Since STEINER did not have the opportunity to learn a foreign language in his youth, he was not able to read specialist articles written in other languages (the compulsory language for dissertations at the Berlin University at this time was still Latin). He left it to JACOBI to find out whether anything had already been published about any new fact he discovered. However, he was not always correct in his citations.

His application to convert his associate professorship into a full professorship was rejected several times for budgetary reasons and it was possible that the fact that he lacked the academic education expected of a university teacher also played a role.

STEINER also proved difficult as a university teacher; contemporaries describe his hot temper, his enthusiastic way of talking about mathematics, but also his sometimes coarse, alienatingly abrupt manners. In the end, he was even at odds with his friend JACOBI.

In the last years of his life, STEINER's health deteriorated visibly and his stays at a health resort for his kidney disease had no effect. He spent most of his time in his Swiss homeland and only during the lecture months did he stay in Berlin.

When he died alone in Bern in 1863, the bachelor left a small fortune, one third of which he bequeathed to the Berlin Academy, and the interest financed the *STEINER Prize* named after him.

A small part of the inheritance was used to hold a regular competition at his old primary school in Utzenstorf and to award prizes to the best mental calculators.

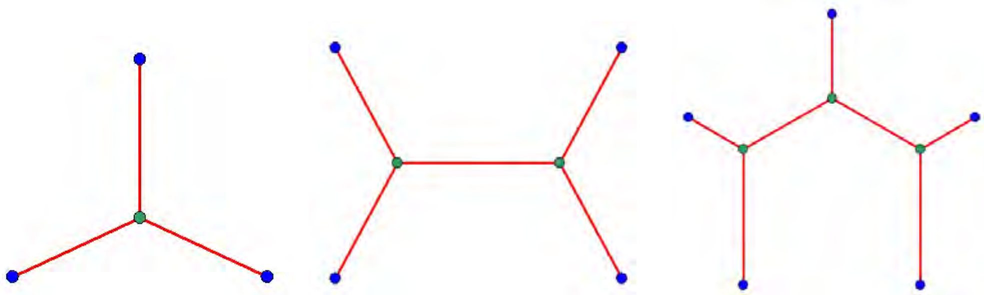
Today, numerous terms still remind us of the topics the ingenious mathematician dealt with:

The **STEINER tree problem** deals with the question of how *n* point graphs can be connected in such a way that the shortest path network is created; *n* – 2 additional points may also be inserted (so called STEINER points).

For *n* = 3 the problem had already been solved by EVANGELISTA TORRICELLI and PIERRE DE FERMAT:

- If all interior angles of the associated triangle are smaller than 120°, then the connecting lines to the additional point each form an angle of 120°.
- If one of the interior angles of the triangle is equal to or greater than 120°, then the sum of the distances to this point is minimal.

The figures show the optimal solutions for the equilateral triangle, for a rectangle and for the regular pentagon.

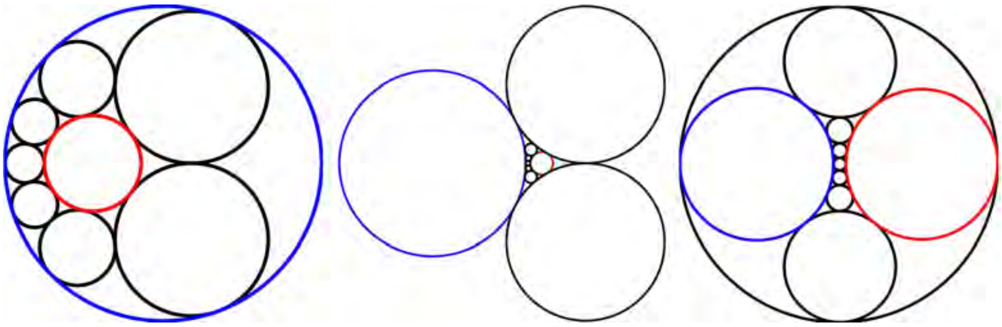


For a general number *n*, the problem is equivalent to one of the 21 classical NP-complete problems, i.e., it is generally hopeless to find an optimal solution in finite computing time.

A (*closed*) **STEINER chain** is a connected sequence of finitely many circles touching each other, which touches two given, non-intersecting circles. STEINER proves the remarkable theorem:

- If such a STEINER chain is possible between two initial circles, then infinitely many can be found. The circles of contact of the circles lie on a circle, the centres of the circles on a conic section.

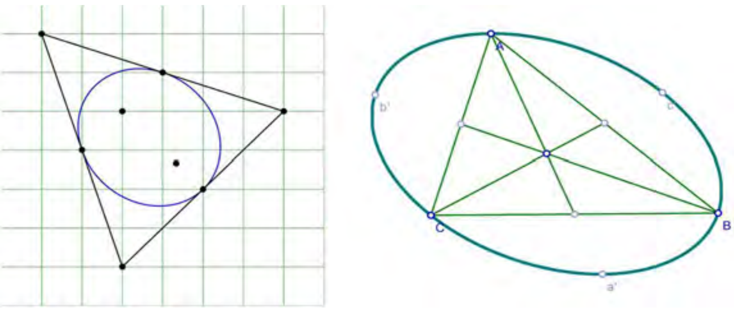
Examples: The initial circles are drawn in red or blue, the circles of the STEINER chain in black and the illustrations show the three possible cases.



(source: Wikipedia)

STEINER ellipses are the ellipses with maximum or minimum area that can be inscribed or circumscribed for a given triangle (the in-ellipse touches the sides in the centre of each side, the centre of gravity of the triangle is the centre of the ellipse).

The following applies to the areas: $A_{\text{inner}} = \frac{\pi}{\sqrt{27}} \cdot A_{\Delta}$ and $A_{\text{outer}} = \frac{4\pi}{\sqrt{27}} \cdot A_{\Delta}$.



(source: Wikipedia)

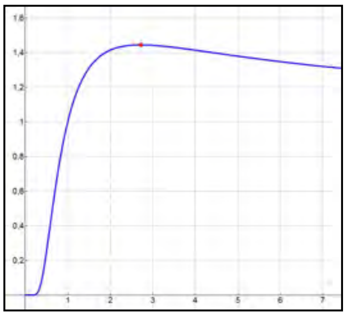
In physics, **STEINER's theorem** is applied when calculating the moment of inertia of a rigid body of mass *m*. If we know the moment of inertia of a rigid body of mass *m*, we can calculate the moment of inertia of a rigid body about a different axis:

- If J_S is the moment of inertia with respect to an axis of rotation which passes through the centre of gravity of the body, then the moment of inertia J_d about an axis of rotation running parallel at a distance *d* can be calculated with the formula $J_d = J_S + m \cdot d^2$.

STEINER's surfaces are special surfaces in space, on the surfaces of which there are families of conic sections. During a joint study visit to Rome in 1844 STEINER (together with DIRICHLET and JACOBI) investigated a special surface, which he called the *Roman surface*, which can be described by the 4th order equation $x^2y^2 + x^2z^2 + y^2z^2 - xyz = 0$.

(source: www.math.tu-dresden.de/modellsammlung/karte.php?ID=349)

Sometimes one finds under the name **Steiner problem** the task of determining the maximum of the function *f* with $f(x) = \sqrt[x]{x}$. This lies at the position *x* = *e*.



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https://www.spektrum.de/wissen/jakob-steiner-1796-1863/1399217

Translated 2021 by John O'Connor, University of St Andrews

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