GREGORY OF SAINT-VINCENT (Sep. 8, 1584 – Jan. 27, 1667)

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Nothing is known about the origin and first years of life of GREGORY OF SAINT-VINCENT (Latin: GREGORIUS A SANCTO VINCENTIO).

He was born in Bruges (today Belgium, then Spanish Netherlands), where he also attended the Jesuit College from 1595.

From 1601 he studied mathematics and philosophy at the university in Douai (today in the Hauts-de-France region), founded by the Spanish KING PHILIP II. In 1605 he entered the Jesuit Order in Rome, first as a novice, then definitively in 1607. He continued his studies at the *Collegio Romano*.

His teacher was CHRISTOPHER CLAVIUS, who was decisively involved in the implementation of the GREGORian calendar reform and was already called the "EUCLID of the 16th century" during his lifetime. He recognised the young man's great mathematical talent and supported him until his death in 1612.

GALILEO's discovery of the moons of Jupiter (1610) also cast doubt on the validity of the geocentric world view in SAINT-VINCENT's mind.

In 1612 SAINT-VINCENT returned to the Spanish Netherlands, finished his theology studies in Leuven (Louvain) and was ordained as a priest.

From 1613 he taught Greek in Brussels and 's-Hertogenbosch. When the first unrest caused by the Dutch independence movement occurred, the presence of Spanish troops in the country was increased and SAINT-VINCENT was seconded to look after the troops.

In Kortrijk (Courtrai) he took the three monastic vows (obedience, chastity, poverty).

During the following years he became a teacher of mathematics at the Jesuit school in Antwerp, then at the university in Leuwen. During this time he published scientific theses on comets and mechanics.

In the 1620s, supported by his best students, Saint-Vincent began to compile materials for his great work *Opus geometricum quadraturae circuli et sectionum coni* (Geometric work on the squaring of the circle and on conic sections), which, however, he would only be allowed to publish with the approval of his superiors in Rome. So he made a request to Mutio Vitelleschi, the Superior General of the Jesuits and he forwarded the submitted manuscripts to Christoph Grienberger, Clavius's successor, so that he could assess the methods that Saint-Vincent had developed. To speed up the matter, Saint-Vincent travelled to Rome, but even after a two-year stay he was unable to get a decision.

In the meantime, two offers had arrived: King Philipp IV of Spain wanted Saint-Vincent to come to Madrid as tutor to his youngest son, and Emperor Ferdinand II offered him the chair of mathematics at the University of Prague, and he wanted him also to act as personal chaplain to the Emperor. (Prague had previously belonged to the sphere of influence of the Protestant camp, but had now been conquered by the imperial troops). Bound by his vow of obedience, Saint-Vincent himself had no choice and his superiors in the order decided to send him to Prague. Before taking up teaching in Prague, Saint-Vincent returned to Leuven once more to put his manuscripts in order, since he wanted to continue editing them in Prague.

However, King Philipp IV did not give up, and in fact succeeded in getting the head of the Jesuit Order to summon Saint-Vincent to Spain, but when the latter suffered a (first) stroke, he remained in Prague for the time being.

In 1631, Prague was conquered by the Saxon troops allied with the Swedish king Gustav II Adolf and a large part of Saint-Vincent's manuscripts were destroyed by fire, but he himself was saved.

With the approval of his superiors, he returned to his native Flanders, where he taught at the Jesuit College in Ghent until the end of his life; despite another stroke, Saint-Vincent reached the – for the time – advanced age of 82.

The manuscripts saved by his students in Prague only came back into his hands ten years later. Thus, after a delay of more than 20 years, his book *Opus geometricum quadraturae circuli et sectionum coni* finally appeared in 1647. Another work on the problem of cube doubling (*Opus geometricum ad mesolabium*) was published posthumously by his students in 1668.

While René Descartes did not see much useful in the two books that could not be written down in one or two pages – a quite typical judgement from Descartes – both Christiaan Huygens and later Gottfried Wilhelm Leibniz praised the author's acumen.

The fact that Saint-Vincent has remained rather unknown as a mathematician is probably due to a serious error he made when he believed he had shown that squaring the circle was possible. Huygens only found the error in the proof after a three-year search, on page 1121 of the book (out of a total of 1250 pages).

This error unjustly overshadowed Saint-Vincent's other achievements. The progress he made was of great importance for the development of the infinitesimal calculus.

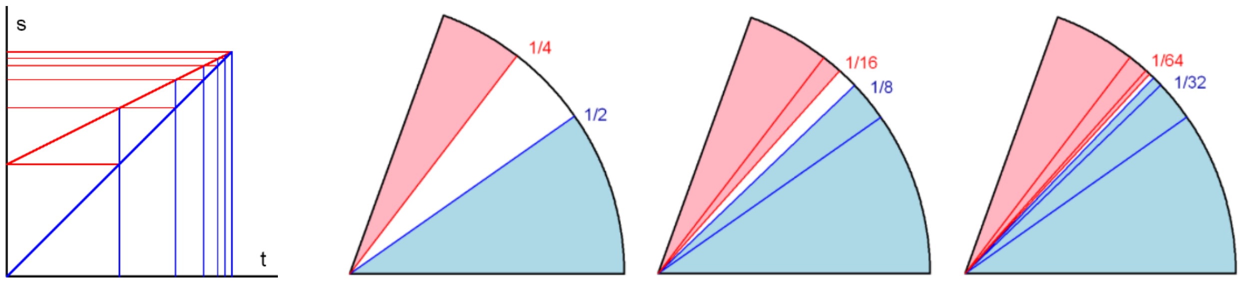
Historians of mathematics also count Saint-Vincent among the "fathers" of analytic geometry.

However, the situation in Central Europe during the 30 Years' War played a crucial role in the fact that his contributions were only printed and became widespread after a long delay.

SAINT-VINCENT was the first to use the term *terminus* for the limit value of a sequence and obviously understood it in this way. That he had grasped the idea of was shown, for example, by the following two problems, which he was the first to solve:

- The so-called *paradox of ACHILLES and the tortoise* is only apparently a paradox, since the running times of the two form a geometrical series and the common limit value determines the time at which ACHILLES overtakes the tortoise (see the following figure on the left).

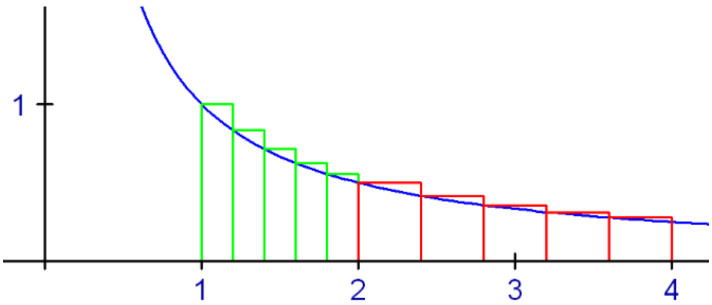
- An *angular trisection* can be done by continuously halving partial angles appropriately, because $\frac{1}{1}-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\ldots-\ldots=\frac{2}{3}$ (figures on the right).



His method of determining volumes, *ductus plani in planum* (subdivision of a surface), is comparable to the method of the indivisibles of BONAVENTURA CAVALIERI (developed independently of the latter and at about the same time). SAINT-VINCENT was the first mathematician to use the term *exhaustion* in this context (from Latin *exhaurire* = to exhaust).

When determining the content of the area below the graph of the hyperbolic function, SAINT-VINCENT discovered a special property. (That this is the characteristic functional equation of the logarithm function was only discovered by his student ALPHONSE ANTONIO DE SARASA.)

In the following diagram (as an example), five overlapping rectangles to the graph of the hyperbolic function *f* with $f(x)=\frac{1}{x}$ on the interval $[1,2]$ as well as on the interval $[2,4]$ (which is twice as large) are entered.



The areas of the rectangles have the same sum, i.e. respectively $0.2\cdot(\frac{1}{1.0}+\frac{1}{1.2}+\frac{1}{1.4}+\frac{1}{1.6}+\frac{1}{1.8})$ and $0.4\cdot(\frac{1}{2.0}+\frac{1}{2.4}+\frac{1}{2.8}+\frac{1}{3.2}+\frac{1}{3.6})$.

Similarly, the equality of areas applies to any interval $[a,b]$ and $[k\cdot a, k\cdot b]$, thus also to the interval $[1,\frac{b}{a}]$ – regardless of the number of rectangles chosen – and in the same way also to smaller rectangles.

Written down in our present notation, this means that for the area contents we have the logarithm property:

$$F(u\cdot v)=\int_1^{u\cdot v}\frac{1}{x}\,dx=\int_1^{u}\frac{1}{x}\,dx+\int_u^{u\cdot v}\frac{1}{x}\,dx=\int_1^{u}\frac{1}{x}\,dx+\int_1^{v}\frac{1}{x}\,dx=F(u)+F(v)\;.$$

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