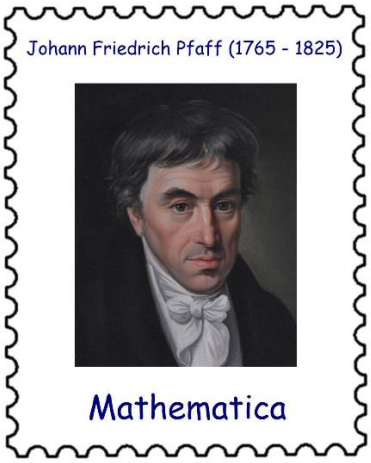
**JOHANN FRIEDRICH PFAFF** (DECEMBER 22, 1765 – APRIL 21, 1825)

by HEINZ KLAUS STRICK, Germany



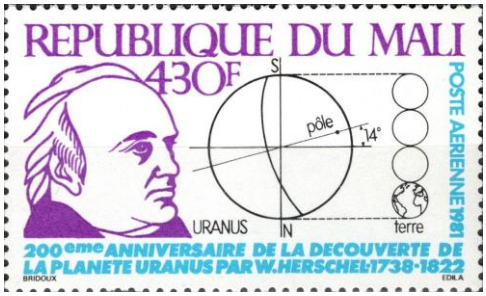

PIERRE SIMON LAPLACE called him the most important mathematician in Germany, and he did not mean CARL FRIEDRICH GAUSS, but JOHANN FRIEDRICH PFAFF.

However, the French scholar made this assessment at a time when the future *Princeps Mathematicorum* had not yet appeared in public. PFAFF is known to only a few people today, but he played an important role in GAUSS's life ...

JOHANN FRIEDRICH PFAFF grew up as the second of seven sons of a high government official in Stuttgart, the capital of Württemberg. His path was marked out by family tradition: at the elite school for sons of the higher classes, the *Hohe Karlsschule*, the future civil servants (administrators, lawyers, officers, medics, builders) were educated with military precision – often in an unbearably authoritarian manner, as we know from FRIEDRICH SCHILLER, who had to attend this school on the orders of Duke CHARLES EUGENE (and against the will of his parents).

Though mathematics played only a minor role during his school education (e.g. in ballistic calculations or the planning of fortifications), the young pupil attracted the attention of his teachers by his extraordinary apprehension. At the age of 15, PFAFF worked independently through EULER's *Introductio in analysin infinitorum*, and at 17, he was appointed *Chevalier* on account of his above-average achievements.

His school attendance ended at the age of 20 without a formal examination procedure, which was unnecessary because of the permanent continuous assessment. For further education, the Duke sent him to Göttingen with a generous scholarship, where PFAFF wrote a paper on astronomy at short notice and won a prize with it. After two years of study with ABRAHAM GOTTHELF KÄSTNER and GEORG CHRISTOPH LICHTENBERG, he went on to Berlin for an astronomy internship with JOHANN ELERT BODE, who had become famous for his work on the orbit of the planet Uranus discovered by WILLIAM and CAROLINE HERSCHEL in 1781.



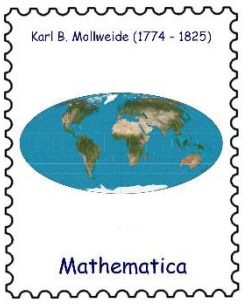
As was his duty, PFAFF kept the Duke's court informed of his activities and conversely, he received his instructions from there. After writing *Commentatio de ortibus et occasibus siderum apud auctores classicos commemoratis* [Commentary on the rising and setting of the stars mentioned by classical authors], PFAFF received his doctorate in 1786 (his supervisors were KÄSTNER and BODE).

When the chair of mathematics at the University of Helmstedt (in the Principality of Braunschweig-Wolfenbüttel) became vacant in 1788, LICHTENBERG urged his former student to apply for the position. First, however, PFAFF had to apply to the Duke to be dismissed from the service of the Duchy of Württemberg.

For the next 22 years – until the closure of the university during the Napoleonic occupation – PFAFF taught at the university (*Academia Julia*), which had been founded in 1576 by the then ruling Duke JULIUS, as the first Protestant university in Northern Germany. His salary was initially comparatively low. Only when he was offered a professorship at the German-speaking Russian university in Dorpat (today: Tartu in Estonia) in 1802 and turned it down, did he finally receive a salary that enabled him to start a family. In his marriage to CAROLINE BRAND, a cousin on his mother's side, two sons were born; the first died early, the second would later publish his father's writings.

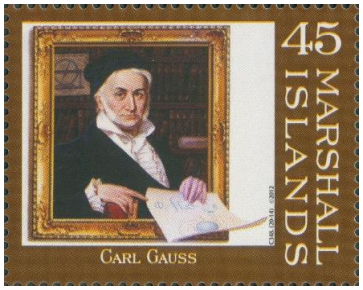
The first student to receive a doctorate supervised by PFAFF in 1796 was CARL BRANDAN MOLLWEIDE, who in 1806 developed the area-preserving map projection named after him (MOLLWEIDE projection, fig. Wikipedia) and found special trigonometric formulae for arbitrary triangles:

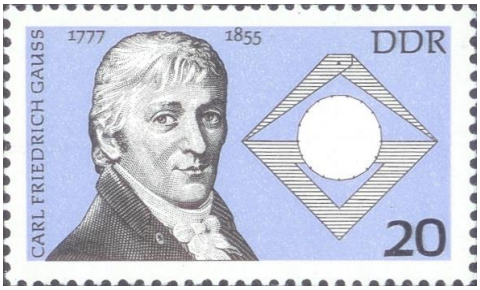
$$\frac{a+b}{c}=\frac{\cos(\frac{\alpha-\beta}{2})}{\sin(\frac{\gamma}{2})} \text{ and } \frac{a-b}{c}=\frac{\sin(\frac{\alpha-\beta}{2})}{\cos(\frac{\gamma}{2})} \text{ (MOLLWEIDE formulas).}$$




PFAFF's most famous student was CARL FRIEDRICH GAUSS, who had come from Göttingen to Helmstedt in 1798 to listen to lectures by PFAFF and at times he even lived in his house. PFAFF quickly recognised the outstanding talent of his new student and encouraged him in all his endeavours.

As the sole examiner of GAUSS's dissertation: *Demonstratio nova theorematis omnem functionem algebraicam rationalem integram unius variabilis in factores reales primi vel secundi gradus resolvi posse* [New proof of the theorem that every whole rational algebraic function in one variable can be decomposed into real factors of the first or second degree], PFAFF supported his student's application to be awarded a doctorate *in absentia*.


In the same year as GAUSS, JOHANN MARTIN CHRISTIAN BARTELS was also awarded a doctorate (supervised by PFAFF, LICHTENBERG and KÄSTNER).

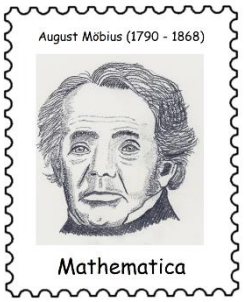

After various activities, BARTELS became a mathematics teacher at JOHANN HEINRICH PESTALOZZI's school, and then a mathematics professor in the Russian city of Kazan. There NIKOLAI LOBACHEVSKY became his best-known student.

As early as 1800, Helmstedt University was threatened with closure, and it was thanks to PFAFF's efforts that the institution remained in existence (for the time being).

After the reorganisation of Germany by NAPOLEON, Helmstedt became part of the Kingdom of Westphalia. Under the government of JÉRÔME BONAPARTE, the university was finally closed in 1810.

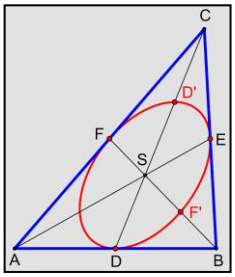
PFAFF was able to transfer to another mathematics chair in the kingdom and thus came to Halle, where he later also became director of the observatory.

Among his students in Halle was AUGUST MÖBIUS, who received his doctorate from him in 1815. (His topic was *On Calculation Methods for Fixed Star Occultations by Planets.*)




As a student, PFAFF had already studied EULER's investigations on series and wrote an essay: *Versuch einer neuen Summationsmethode* [Attempt at a new summation method], which he dedicated to his Duke.

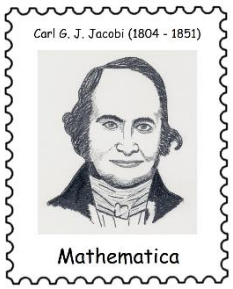
In 1797 he published the first volume of a textbook on analysis (why a second volume was never produced is not known) – in Latin and in the style of his great role model GAUSS: *Disquisitiones analyticae maxime ad calculum integralem et doctrinam serierum pertinentes* [Analytical investigations mainly related to the integral calculus and the series theory].



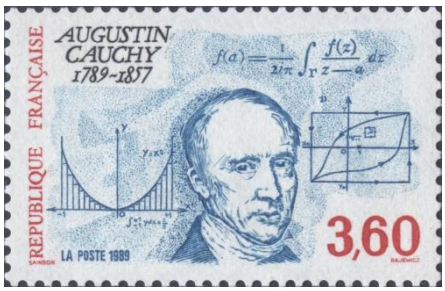

In 1810 PFAFF solved a problem posed by GAUSS: *How can an ellipse with maximum area be inscribed in an arbitrary triangle or in a convex quadrilateral*?

In 1815 he published his most important contribution in the Proceedings of the Berlin Royal Academy: *Methodus generalis, aequationes differentiarum partialium, necnon aequationes differentiales vulgares, utrasque primi ordinis, inter quotcunque variabiles complete integrandi* [General method of completely integrating partial differential equations and ordinary differential equations, both of the first order, in any number of variables]. A partial differential equation is an equation in which a relationship between a function with several variables and its partial derivatives (i.e. the derivatives by the individual variables, in which the remaining variables are left constant in each case). PFAFF introduced a special notation in this treatise, which is still called the *PFAFFian form* today.

GAUSS wrote an extremely positive review on this for the Göttingen Academy of Sciences and made clear what progress had been made compared to the approaches of JOSEPH-LOUIS LAGRANGE. However, it was not until 1827 that CARL GUSTAV JACOB JACOBI took up PFAFF's ideas and developed them further.






JOHANN FRIEDRICH PFAFF was an important mathematician in the line of succession of EULER and LAGRANGE and his achievements were recognised by his membership of various academies (Göttingen, Berlin, Paris, St Petersburg).

The new beginning of analysis through the exactness requirements of AUGUSTIN-LOUIS CAUCHY took place during his lifetime, but PFAFF had no time to react to it.

He died unexpectedly at the age of 59 – shortly before the beginning of the summer semester for which he had already announced his lectures.

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https://www.spektrum.de/wissen/johann-friedrich-pfaff-1765-1825-einer-der-bedeutendsten-mathematiker-deutschlands/1628022

Translated 2023 by John O'Connor, University of St Andrews

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