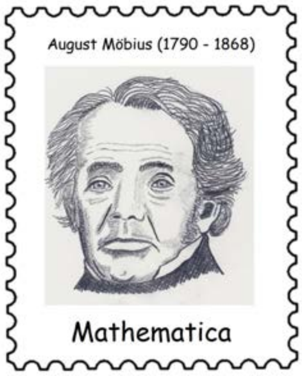
AUGUST FERDINAND MÖBIUS (November 17, 1790 – September 26, 1868)

by HEINZ KLAUS STRICK, Germany



His name is inextricably linked to a figure that can be drawn as a one-sided surface that requires twice as much paint to colour as one might have thought. It was thus that AUGUST FERDINAND MÖBIUS himself characterised the object that later became known as the MÖBIUS strip or MÖBIUS band. This geometrical object was discovered independently in 1858 by MÖBIUS and the Göttingen professor of mathematics JOHANN BENEDICT LISTING (1808 – 1882). (drawings © Andreas Strick)

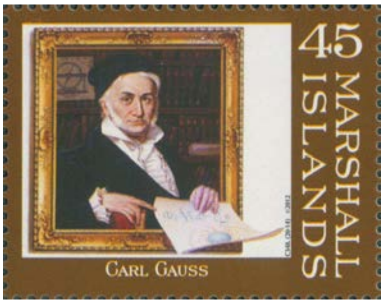
It was only a few years previously that one could read in a popular geometry text by CHRISTIAN VON STAUDT what seemed a perfectly reasonable characterisation of a surface embedded in three-dimensional space: every surface has two sides. Yet the MÖBIUS strip is a surface that fails to possess that characteristic: there is no way to distinguish front and back, or top and bottom.

It is difficult to figure out today why the strip was named after MÖBIUS and not after LISTING, despite the fact that as early as 1862, he had publicized the properties of this figure – which has only one edge and one face – within the mathematical community in his article *Der Census räumlicher Complexe oder die Verallgemeinerung des EULER'schen Satzes von Polyedern* (A census of spatial complexes, or a generalization of EULER's theorem on polyhedra). MÖBIUS had unsuccessfully submitted his 1861 paper *Über die Bestimmung des Inhalts eines Polyeders* (On the determination of the volume of a polyhedron) to the French Academy, and his article on one-sided surfaces and polyhedra for which his "law of edges" fails to hold appeared only in 1865.

A MÖBIUS strip appeared in two Brazilian postage stamps on the occasion of the International Congresses of Mathematicians in 1967 and 1973, and it is also used to signify an "unending banding together", as, for example, among the Benelux countries.

AUGUST FERDINAND was the only child of a dance instructor at the royal state school in Schulpforte (near Naumburg). After the early death of his father, the boy's mother, a descendent of MARTIN LUTHER, took over her son's education. At age 13, AUGUST attended the state school, where he showed a particular interest in mathematics.

In 1809, he gratified his family's wish by undertaking legal studies at the University of Leipzig, but he soon switched to mathematics, astronomy, and physics. He was influenced particularly by the professor of astronomy KARL MOLLWEIDE, who is known as the inventor of an equal-area cartographic projection and certain trigonometric formulas. He also spent time in Göttingen, where he attended lectures given by CARL FRIEDRICH GAUSS, and in Halle, where he heard lectures by JOHANN FRIEDRICH PFAFF, one of GAUSS's teachers.



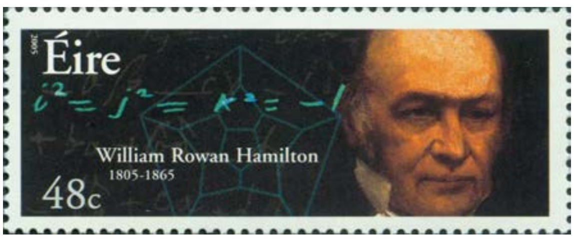
In 1815, he completed his doctoral dissertation on the calculation methods for occultations of fixed stars by planets (*De computandis occultationibus fixarum per planetas*), followed by his habilitation thesis on trigonometric equations. Since MOLLWEIDE had in the interval switched his area of research from astronomy to mathematics, MÖBIUS hoped to assume the vacated chair in astronomy. And in fact, in 1816, he was named associate professor of astronomy and mechanics at Leipzig. At the same time, he was given the responsibility as "observer" of overseeing the reconstruction of the observatory. At the beginning of his tenure in Leipzig, he had difficulties in presenting his lectures, so that he was unable to attract a sufficient number of paying students.

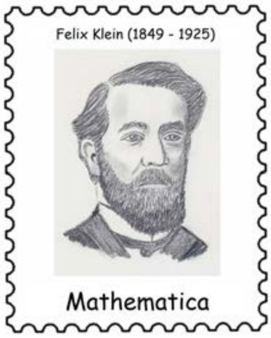
Soon, MÖBIUS was offered a professorship in mathematics at Greifswald, and also a chair in astronomy at the German university in Dorpat (today Tartu, Estonia). He declined both of these offers in the hope that his loyalty to the kingdom of Saxony would be rewarded. When MOLLWEIDE died in 1825, however, his chair in mathematics went to another person. It was not until 1844, when the University of Jena attempted to recruit him, that he finally obtained the long sought full professorship of higher mechanics and astronomy in Leipzig.

In the meantime, he published a number of books on astronomical subjects that gained considerable recognition, as well as a textbook on statics. He allowed himself considerable time for preparing his publications, so that he could lay aside his completed manuscripts for a time in order to reconsider everything he had written before submitting the text for publication.

From 1840 on, he worked on topological questions. A problem that he posed became particularly well known, namely the five-princes problem: *Once upon a time, there was a king who had five sons. Since he could not decide which of his sons should be his successor, he decided that on his death, his kingdom would be divided into five parts in such a way that each region would share a border with each of the other regions. Is it possible to satisfy this condition?*

Problems of this nature were taken up by other mathematicians only several years later. For example, the four colour problem was formulated by FRANCIS GUTHRIE in the year 1852, and later investigated by AUGUSTUS DE MORGAN and WILLIAM ROWAN HAMILTON.



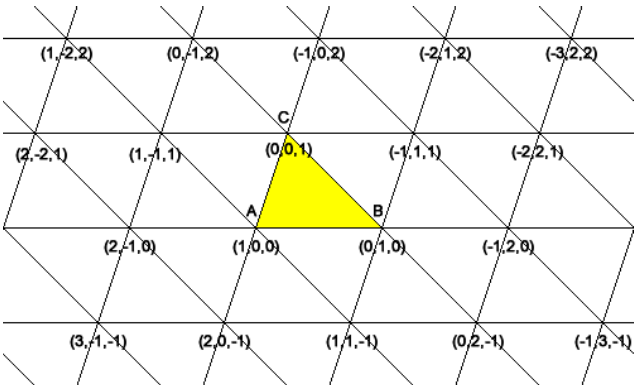


MÖBIUS's most important mathematical work appeared in 1827, namely *Der barycentrische Calcul – ein neues Hilfsmittel zur analytischen Behandlung der Geometrie* (The barycentric calculus – a new aid in the analytic treatment of geometry). In this work, he develops the "theory of geometric relationships among figures" (congruence, similarity, affinity, collineation), which later was used by FELIX KLEIN in his classification of various geometric approaches (his *Erlangen Programme*).

In his inaugural lecture in 1872, KLEIN said, "Today, we rightly honour AUGUST FERDINAND MÖBIUS as one of the ground-breaking geometers of the nineteenth century, whose work has influenced the development of mathematics even into our own time."

MÖBIUS introduced homogeneous coordinates and in doing so, introduced a principle taken from astronomy, namely that of replacing a group of masses distributed at various points throughout space by a single body at the centre of mass. Thus every point in the interior of a triangle ABC can be represented as the centroid if one suitably weights the vertices.

The barycentric (Greek βαρύς = heavy) coordinates in a plane relate to a triangle ABC whose vertices have the coordinates $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.



(source: Wikipedia, Wfstb)

For each point P in the plane, one has the relationship $P = \alpha \cdot A + \beta \cdot B + \gamma \cdot C$, where $\alpha + \beta + \gamma = 1$. For example, the midpoints of the sides of the triangle have barycentric coordinates $(\frac{1}{2}, \frac{1}{2}, 0)$, $(\frac{1}{2}, 0, \frac{1}{2})$ and $(0, \frac{1}{2}, \frac{1}{2})$, while the intersection of the medians has coordinates $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

The name MÖBIUS is attached to certain mappings of the extended complex plane $\mathbb{C} \cup \{\infty\}$ to itself (MÖBIUS transformations). They are defined by the rule $z \mapsto \frac{az+b}{cz+d}$, with $a, b, c, d \in \mathbb{C}$ and $ad - bc \neq 0$, and they can be described as the result of a sequence of translations, rotations, and inversions.

In 1831, MÖBIUS wrote his article *Über eine besondere Art von Umkehrung der Reihen* (On a certain type of inversion of series). Beginning with a function *f* of the form $f(x) = a_1 x + a_2 x^2 + a_3 x^3 + \ldots$, he seeks to determine coefficients $b_1, b_2, b_3, \ldots$ such that $x = b_1 \cdot f(x) + b_2 \cdot f(x^2) + b_3 \cdot f(x^3) + \ldots$

In investigating the properties of the coefficients, he introduced a certain function μ (known today as the MÖBIUS function); it also plays an important role in number theory and combinatorics.

It is defined for natural numbers *n* by $\mu(1) = 1$ and by $\mu(n) = (-1)^k$ if *n* is the product of *k* unique prime factors (i.e., *n* is square-free), and $\mu(n) = 0$ otherwise. It is obvious that for a prime number *n*, one has $\mu(n) = -1$.

The function μ is multiplicative for relatively prime integers a, b; that is, $\mu(a \cdot b) = \mu(a) \cdot \mu(b)$.
If one sums the values μ(d) for all divisors d of n (for $n > 1$), then the result is always 0.

In his later years, MÖBIUS became a corresponding member of a number of academies, which recognized his outstanding scientific contributions. He kept up his duties as university lecturer and director of the observatory until his death.

With his wife, DOROTHEA ROTHE, he had three children. One of his grandchildren, PAUL JULIUS MÖBIUS, a neurologist and lecturer in Leipzig, made important contributions to mental disorders. Indeed, SIGMUND FREUD referred to him as one of the founders of psychotherapy. He achieved, however, a dubious notoriety through his 1900 scientifically indefensible publications *Über die Anlage zur Mathematik* (On the aptitude for mathematics) and *Über den physiologischen Schwachsinn des Weibes* (On the Physiological Idiocy of Women).

First published 2013 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/august-ferdinand-moebius-1790-1868/1183271

Translated by David Kramer

English version first published by the *European Mathematical Society* 2014

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".