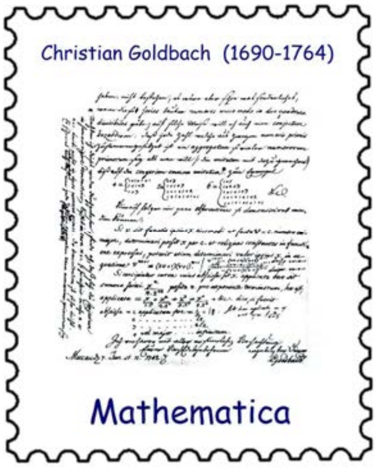
CHRISTIAN GOLDBACH (March 18, 1690 – November 20, 1764)

by HEINZ KLAUS STRICK, Germany



One of the most famous, still unproven conjectures of number theory is:

- *Every even number greater than 2 can be represented as the sum of two prime numbers.*

The scholar CHRISTIAN GOLDBACH made this simple mathematical statement to his pen pal LEONHARD EULER in 1742 as an assumption.

(In the original version it said: *Every natural number greater than 2 can be represented as the sum of three prime numbers*, since at that time the number 1 was still considered a prime number.)

All attempts to prove this theorem have so far failed. Even the offer of a prize of one million dollars hardly led to any progress. CHEN JINGRUN (1933-1996, Chinese stamp on the left), student of HUA LUOGENG (1910-1985, stamp on the right), the most important Chinese mathematician of the 20th century, succeeded in 1966 in making the "best approximation" to GOLDBACH's conjecture. CHEN JINGRUN was able to prove that any sufficiently large even number can be represented as the sum of a prime number and another number that has at most two prime factors.

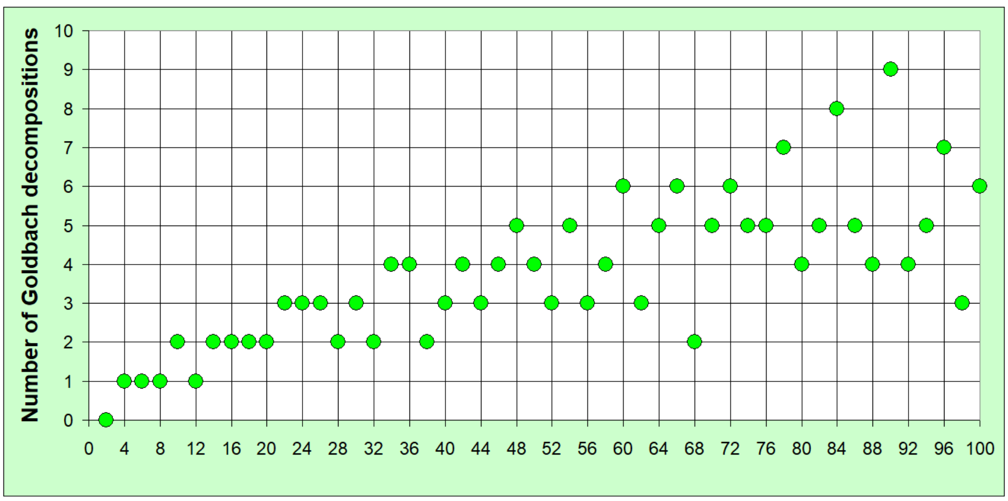
Among the first even numbers are those that have only *one* GOLDBACH decomposition:

4 = 2 + 2; 6 = 3 + 3; 8 = 3 + 5; 12 = 5 + 7.

For larger even numbers one finds a "tendency" to increase the number of possibilities, but even then there is always a number that has only a few decompositions, such as

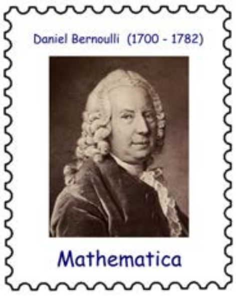
98 = 19 + 79 = 31 + 67 = 37 + 61.

See the graph below and *The On-Line Encyclopedia of Integer Sequences* A045917.



CHRISTIAN GOLDBACH, son of a protestant pastor, grew up in Königsberg (East Prussia, today Kaliningrad) and attended a secondary school and university there. During his studies he was mainly engaged in law and medicine. Longer study trips between 1710 and 1724 led him to many European cities, where he met many important mathematicians:

In Leipzig he visited GOTTFRIED LEIBNIZ, in London he exchanged ideas with ABRAHAM DE MOIVRE, in Oxford he met NICOLAUS BERNOULLI (I) and in Venice his cousin NICOLAUS II, who made contact with his younger brother DANIEL (all nephews of JACOB and JOHANN BERNOULLI).



(drawing © Andreas Strick)

Returning to Königsberg in 1724, he met two scholars who were travelling through: the German philosopher GEORG BERNHARD BILFINGER and the Swiss mathematician JAKOB HERMANN, who were on their way to St Petersburg to build up an *Academy of Sciences* there, following the Berlin example.

The following year GOLDBACH applied to the president of the new *Academy* for a post, but was initially rejected. At the end of 1725 he was appointed to a chair of mathematics and history at the *Academy*.

During his student days, GOLDBACH had hardly been interested in mathematics. However, since his encounter with LEIBNIZ, his interest in mathematical topics had increased, as shown, for example, by a contribution on infinite series in the *Acta eruditorum*.

After the founding ceremony of the *Academy*, GOLDBACH took over the office of secretary and carried out this coordinating function until 1727, when he was appointed teacher to the young Tsar PETER II (a grandson of PETER THE GREAT). Tsarina CATHERINE I had decreed that her grandson, only 12 years old, should follow her on the tsar's throne.

In the struggle for the real power in the country between the rival generals MENSCHIKOV and DOLGORUKOV, Moscow temporarily became the capital of Russia again, so that GOLDBACH had to move with the court. When the young tsar died five years later, GOLDBACH remained in Moscow until the new tsarina ANNA IVANOVNA moved the court back to St Petersburg in 1732.

After ANNA IVANOVNA's death in 1740 her son, just a few weeks old, was temporarily proclaimed tsar until ELISABETH, a daughter of PETER THE GREAT, seized power.

CHRISTIAN GOLDBACH - as one of the few at court – survived all these changes of government without damage.

GOLDBACH had less and less time to devote to mathematics. In 1729 and then again in 1732 he published an article on infinite series. His burden of administrative tasks within the management of the Academy grew from year to year, until he finally asked for a reduction of his duties.

In 1740 GOLDBACH was even completely relieved of his academy duties, since the new tsarina promoted the eloquent cosmopolitan to an important post in the Foreign Ministry. In the following years this helped him to acquire great wealth and land ownership.

Mathematics remains his favourite pastime, and in LEONHARD EULER he had a highly competent correspondent.

LEONHARD EULER and CHRISTIAN GOLDBACH had met personally in 1727 when EULER began teaching in St Petersburg. During GOLDBACH's time in Moscow the two scholars began a lively correspondence which they continued for 35 years. The internal political turbulences of the years 1740/1741 led EULER to accept a call to Berlin, where he took up the post of director of the mathematical class of the *Prussian Academy of Sciences*.

It was mainly on problems of number theory that the two exchanged views. GOLDBACH was not only concerned with the above-mentioned conjecture. Through his investigations he gave many suggestions to EULER, who was able to solve many of these problems:

- Representability of odd natural numbers: GOLDBACH conjectured that every odd natural number (greater than 17) could be represented in the form $2 \cdot n^2 + p$ where *p* is a prime number: $19 = 2 \cdot 1^2 + 17 = 2 \cdot 2^2 + 11$; $21 = 2 \cdot 1^2 + 19 = 2 \cdot 2^2 + 13 = 2 \cdot 3^2 + 3$; $23 = 2 \cdot 3^2 + 5$; $25 = 2 \cdot 1^2 + 23 = 2 \cdot 2^2 + 17 = 2 \cdot 3^2 + 7$; $27 = 2 \cdot 2^2 + 19$; $29 = 2 \cdot 3^2 + 11$; …

  EULER examined the odd numbers up to 999 and GOLDBACH checked the conjecture even up to the number 2499.

  MORITZ STERN found two counterexamples (5777 and 5993) in 1856. It is not known whether there are any other counterexamples.

- Properties of *FERMAT numbers* (natural numbers of the form $F_n = 2^{2^n} + 1$ which FERMAT assumed were always prime numbers). EULER found out in 1732 that $F_5 = 4{,}294{,}967{,}297$ is not prime, for the number is divisible by 641.

  Today it is believed that only the numbers F_0 to F_4 are prime numbers.

- Properties of *MERSENNE numbers* (natural numbers of the form $M_n = 2^n - 1$) and of *perfect numbers* (natural numbers whose sum of the real divisors is as great as the number itself):

  EUCLID had already shown that every natural number of the form $2^{n-1} \cdot (2^n - 1)$ is perfect if $2^n - 1$ is a prime number; EULER proved that the converse of the proposition is also true: every *even* perfect number is of this form.

- *Prime-generating polynomials*: In 1772 EULER found the polynomial $n^2 + n + 41$ which, when the natural numbers $n = 0, 1, 2, 3, ..., 39$ are inserted, results in prime numbers.
- Representability of natural numbers as the sum of square numbers, cubes, general *k*-th powers, determination of the smallest number $g(k)$ of necessary summands, where $g(2) = 4$ (this is the so-called *LAGRANGE's 4-squares theorem*), $g(3) = 9$, $g(4) = 17$, $g(5) = 37$ (proved by CHEN JINGRUN in 1964).

  This generalization is called WARING's problem (after EDWARD WARING, 1736-1798).

GOLDBACH did not live to see EULER's return to Russia (1766). In 1764 the highly esteemed scholar and statesman died in Moscow at the age of 74.

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https://www.spektrum.de/wissen/christian-goldbach-der-mann-der-die-primzahlen-liebte/1627116

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Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".