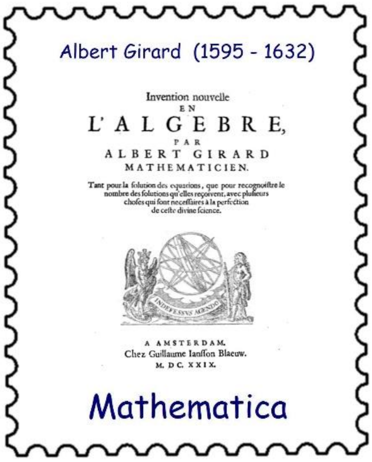
ALBERT GIRARD (1595 – December 8, 1632)

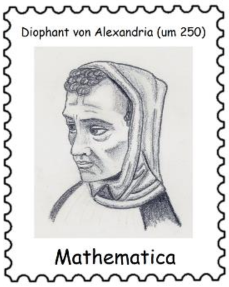
by HEINZ KLAUS STRICK, Germany



ALBERT GIRARD spent the first years of his life in Saint-Mihiel (south of Metz, then part of the Duchy of Lorraine). In 1610, his family had to leave their homeland because, despite the tolerant regulations of the *Edict of Nantes* (1598), the members of the Reformed Church in the Lorraine territories under French rule were forbidden to hold services according to their rites. These Protestants found refuge in Amsterdam, where ALBERT GIRARD married at the age of 18 and started a family. In 1615 his first child was born and the eleventh was born in 1632, a few weeks after GIRARD's early death.

In the first years of his exile, ALBERT GIRARD performed as a professional lute player. He exchanged ideas on mathematical problems with the mathematician JACOB GOLIUS. From 1617 onwards he studied music and mathematics at the University of Leiden and he then entered the service of FRIEDRICH HEINRICH VON ORANIEN, governor of the United Netherlands, son of WILHELM I and father of WILHELM II OF ORANGE, as an engineer.

As part of this job, he was mainly involved in the construction of fortifications for his new homeland, which was fighting for its independence from Spain. In a letter, the diplomat CONSTANTIJN HUYGENS, father of CHRISTIAAN HUYGENS, confirmed the great commitment and skills of the engineer.



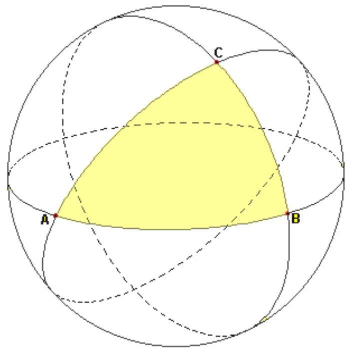
(drawings © Andreas Strick)

In 1625, the *Samielois* (from Saint Mihiel) GIRARD published his first work, the annotated and expanded version of SIMON STEVIN's *Arithmétique*. In doing so, he attached great importance to clearly separating STEVIN's original texts from his own comments, because he did not want to *"... mix what the author says with what I write*". He supplemented the work with the translation of Books V and VI of the *Arithmetica* of the DIOPHANTUS OF ALEXANDRIA.

Also in 1625, GIRARD's French translation of a book on fortress construction, originally written in Flemish, was published by the publisher HENDRIK HONDIUS from La Haye (the Hague).

(The latter was *not* related to the publisher JODOCUS HONDIUS from Ghent, who had bought the printing plates of GERHARD MERCATOR's World Atlas in 1604 and published it as an extended World Atlas in 1606. Occasionally one finds the portrait of JODOCUS HONDIUS with the false indication that it is a portrait of ALBERT GIRARD).

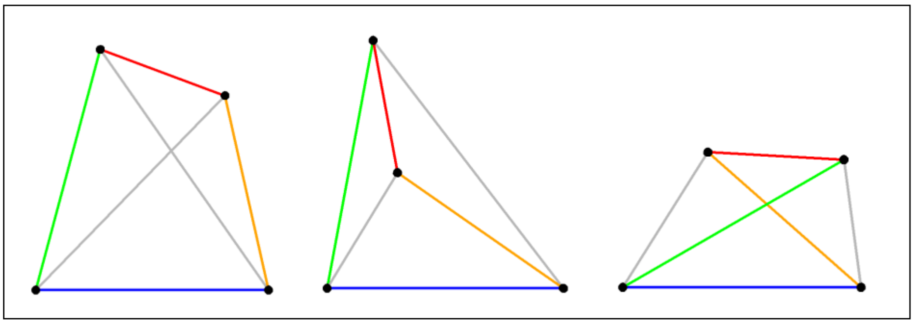
In 1626, GIRARD published a corrected version of STEVIN's work on trigonometry tables, in which he was the first in the history of mathematics to use the abbreviations sin, tan and sec ($= \frac{1}{\cos}$). The work also contained a compilation of formulae for calculating quantities in plane and spherical triangles, including the theorem on how to calculate the area *A* of a spherical triangle formed by great circles from the radius *r* of the sphere and the angles *α, β* and *γ* (given in radians):



$A = r^2 \cdot (\alpha + \beta + \gamma - \pi)$ (GIRARD's theorem). (Fig. Wikipedia)

The factor $\alpha + \beta + \gamma - \pi$ (the so-called *spherical excess*) indicates by how much the sum of the interior angles exceeds the value of 180°, i.e. the sum of the angles in the plane triangle. In his main work published in 1629 (see below), GIRARD generalised the above-mentioned area formula for any spherical polygons (today this is known as GAUSS's formula).

The book on trigonometry also contained explanations about the possible shapes of quadrilaterals, pentagons and hexagons, if the side lengths are given in each case:



He distinguished three forms of quadrilaterals, namely convex ones with two diagonals running inside the quadrilateral (cf. fig. left), concave (inverted) quadrilaterals with one diagonal running inside and one outside (cf. middle fig.), and overlapping quadrilaterals with both diagonals outside the quadrilateral (cf. fig. right).

For the pentagons, he gave eleven different shapes, namely four convex or concave, four with two overlaps (which therefore consist of two sub-areas) and one shape each in which the overlaps create three, four or six sub-areas.

In the case of the hexagons, he arrived at 69 different types (actually there are 70).

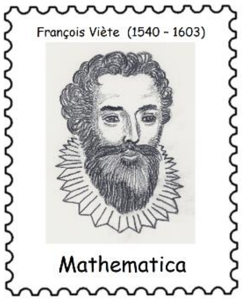
In this work, GIRARD also considered the special case of (convex) cyclic quadrilaterals, of which, for given side lengths *a, b, c, d*, there are only three variants with regard to the order of the sides (*abcd, abdc, acbd*); he designated the diagonals occurring in these quadrilaterals (over the sides *a* and *b* or *a* and *c* or *a* and *d*) as δ_1, d_2 and d_3 and then developed the remarkable formula for the area $A = \frac{\delta_1 \cdot \delta_2 \cdot \delta_3}{4r}$.

GIRARD's major work, *Invention Nouvelle en l'Algèbre*, published in 1629 and comprising 64 pages, rightly bore the term *Invention* in its title, for it contained a number of new results, including the theorem that we now call the *Fundamental Theorem of Algebra*:

> *Toutes les équations d'algèbre reçoivent autant de solutions, que la denomination de la plus haute quantité le demonstre.*

(Each algebraic equation contains as many solutions as the highest exponent indicates).

In contrast to his predecessors SIMON STEVIN and FRANÇOIS VIÈTE, GIRARD no longer limited himself to the consideration of real solutions, but realised that the "impossible" solutions (complex numbers) must also be counted; calculating with these numbers was no trouble at all for him.



GIRARD (latinised: ALBERTUS GERARDUS METENSIS) was also the first to understand the connection between zeros and coefficients of a polynomial:

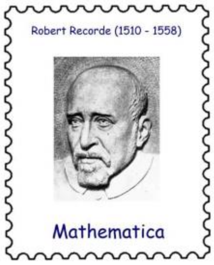
For example, if the equation ordered by even and odd powers is $x^4 - 7x^2 - 24 = 4x^3 - 34x$ with the solutions $-3, +1, +2, +4$, then the coefficient of x^3 results from the sum of the solutions ($-3+1+2+4=4$), that of x^2 from the sum of the six products of two solutions each ($(-3)\cdot 1 + (-3)\cdot 2 + (-3)\cdot 4 + 1\cdot 2 + 1\cdot 4 + 2\cdot 4 = -7$), the coefficient of x from the sum of the four products of three solutions each ($(-3)\cdot 1\cdot 2 + (-3)\cdot 1\cdot 4 + (-3)\cdot 2\cdot 4 + 1\cdot 2\cdot 4 = -34$) and the constant term from the product of the four solutions $(-3)\cdot 1\cdot 2\cdot 4 = -24$.

Corresponding properties also apply if solutions occur more than once or are complex numbers. He showed this by the example $x^4 + 3 = 4x$ with the solutions $+1, +1, -1-\sqrt{-1}, -1+\sqrt{-1}$.

Further he showed: If the coefficients of the powers of x (with decreasing exponents) are $A, B, C, D, \ldots$ then A is equal to the sum of the solutions (see above), $A^2 - 2B$ is equal to the sum of the squares of the solutions, $A^3 - 3AB + 3C$ is equal to the sum of the third powers of the solutions, $A^4 - 4A^2B + 4AC + 2B^2 - 4D$ is equal to the sum of the fourth powers, etc.

GIRARD introduced new notations for algebraic terms. He was the first to mark connected terms by placing round or square brackets; for the product of terms, he omitted the multiplication point.

He wrote down some powers by putting the exponent in brackets and placing it in front of the base (e.g. $(\frac{3}{2})\,49$ stands for $49^{3/2} = 343$); but he was also the first to use the root sign with the root exponent in superscript (e.g. $\sqrt[3]{}$).



In the case of equality of terms, he wrote "*égale*" (French for equal); he did not yet use the sign "=". (This was introduced as early as 1557 by the Welsh mathematician ROBERT RECORDE, but it had only been customary on the continent since LEIBNIZ).

GIRARD died at the young age of 37, leaving behind a penniless widow with eleven children. His widow saw to it that his last work was published posthumously: an annotated and expanded complete edition of SIMON STEVIN's works. The book contained, among other things (for the first time in the history of mathematics), the recursion formula $f_{n+2} = f_{n+1} + f_n$ for the now so-called FIBONACCI numbers as well as approximate fractions for $\sqrt{2}$ and $\sqrt{10}$, which suggest that GIRARD was on the verge of the discovery of their continued fraction development.

First published 2020 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/albert-girard-1595-1632-vater-von-elf-kindern/1741334

Translated 2022 by John O’Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".