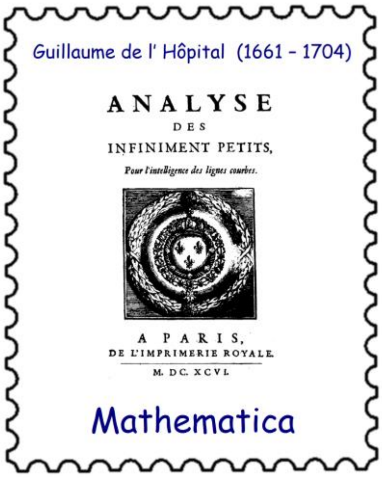
GUILLAUME DE L'HÔPITAL (1661– February 2, 1704)

by HEINZ KLAUS STRICK, Germany



An important theorem of the differential calculus has entered mathematical literature under the name L'HÔPITAL's Rule.

It is named after GUILLAUME FRANCOIS ANTOINE DE L'HÔPITAL, Marquis de Sainte Mesme, Comte d'Entremont, Seigneur d'Oucques, La Chaise, Le Breau and other places, who – as can be inferred from the noble titles – came from one of the most influential French aristocratic houses.

The rule basically says that for differentiable functions f and g which both have a zero at the point $x = a$ you can find the limit of the quotient function $\frac{f}{g}$ with the aid of the value of the function $\frac{f'}{g'}$ if this exists. More precisely:

- *If* $f(a) = g(a) = 0$ *and* $\lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$ *exists, then* $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$.

L'Hôpital published this theorem in 1696 in his book *Analyse des infiniment petits pour l'intelligence des lignes courbes* (Infinitesimal calculus with application to curved lines), the first book on LEIBNIZ's differential calculus. But whether this rule was actually discovered by L'HÔPITAL is disputed and will never be known for certain.

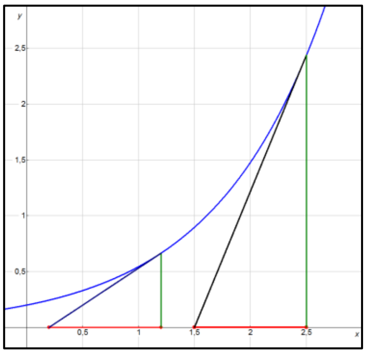
Even as a child, L'HÔPITAL was very interested in mathematical problems. As a 15-year-old, he was even said to have solved a difficult problem about the cycloid (the curve traced by a point on a circle rolling on a line) posed by BLAISE PASCAL.

In accordance with family tradition, he entered military service, leading a cavalry regiment as a captain – but his interest was more in geometry. Due to extreme short-sightedness, he soon had to end his military career. From then on, he regularly attended the philosophical-mathematical discussion group of the mathematics professor NICOLAS MALEBRANCHE from the Order of the Oratorians.

When the 24-year-old JOHANN BERNOULLI came to Paris in 1691, L'HÔPITAL finally found the person who was familiar with the new infinitesimal calculus.

JOHANN BERNOULLI had actually already decided to study medicine at the University of Basel when his attention was drawn to GOTTFRIED WILHELM LEIBNIZ's writings on calculus by his brother JACOB, who was 12 years his senior. Initially under his brother's guidance, but then increasingly on his own, he familiarised himself with the new methods and became his brother's rival. In 1691 he lectured in Geneva and then travelled on to Paris.

JOHANN BERNOULLI agreed to give four lectures a week on the infinitesimal calculus to the members of the MALEBRANCHE circle. He then continued this in return for payment at the L'HÔPITAL's country house in Oucques (Loire). After his return to Basel, he regularly sent further "lessons" in the form of letters.



In 1639, the amateur mathematician FLORIMOND DE BEAUNE had asked DESCARTES for which curves the subtangents (projections of the tangents onto the abscissa) are constant, but neither DESCARTES nor FERMAT could answer this question.

In 1692, in a letter to HUYGENS, L'HÔPITAL informed him of the solution to the problem (the property applies to all exponential functions), without explicitly stating that it did not originate from him but from JOHANN BERNOULLI. He also published the solution under a pseudonym.

JOHANN BERNOULLI was incensed by this and broke off his contacts with L'HÔPITAL until the latter made him an extremely generous financial offer:

JOHANN BERNOULLI should continue to inform him regularly about new mathematical findings in return for half a professor's salary, but he should also undertake not to communicate these new insights to others (L'HÔPITAL expressly mentioned the mathematician PIERRE DE VARIGNON, with whom JOHANN BERNOULLI was friends) or even to publish them.

JOHANN BERNOULLI accepted the offer, but was nevertheless disappointed when he read in 1696 in the preface of the book *Analyse des infiniment petits*:

> *Finally, I am indebted to Messrs BERNOULLI for their great insights, especially to the younger Mr BERNOULLI, who is now professor at Groningen. I have made unceremonious use of their discoveries, as well as those of Herr LEIBNIZ. Therefore, I agree with everything they may claim as their idea, and am satisfied with what they leave me.*

JOHANN BERNOULLI adhered to the agreement he made with L'HÔPITAL to keep quiet about his part in the book; but after the latter's death, he increasingly spread the word that he was the real author of this extraordinarily successful book.

Since he had repeatedly asserted various claims of priority over the years (especially over his older brother JACOB), this was not considered credible for a long time. Only when transcripts of JOHANN BERNOULLI's lectures were found in 1921 did they show great correspondence with the remarks in the book. On the other hand, the lecture notes contain several errors that L'HÔPITAL had obviously recognised and corrected before publication.

Even if it is no longer possible to clarify what part L'HÔPITAL himself played in the book *Analyse des infiniment petits pour l'intelligence des lignes courbes*, his merits should not be underestimated. Without a doubt, the clarity of the structure and the comprehensibility of the formulations contributed to the successful dissemination of LEIBNIZ's theory of the infinitesimal calculus.

The book began with two definitions and a first corollary:

- *Variable quantities are those that continuously increase or decrease, as opposed to constant quantities, which remain unchanged while the others change.*

- *The infinitesimal part by which a variable quantity becomes larger or smaller is called the differential of the quantity.*
- *It is obvious that constant quantities have differential zero.*

Then two axioms followed:

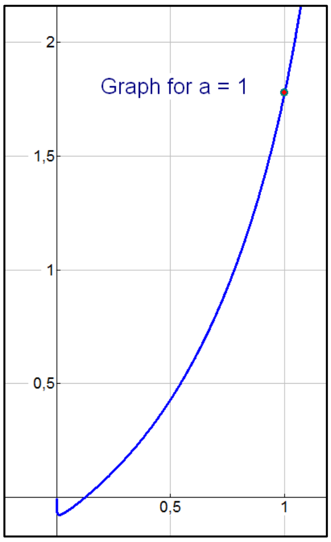
- *If two variable quantities differ only by an infinitesimal amount, then they are said to be equal.*
- *A curved line (curve) is to be regarded as a union of infinitely many infinitesimally small straight lines, or what is the same: as a polygon with infinitely many sides, each of infinitesimally small length.*
- *The angle between two consecutive sides determines the curvature of the curve.*

Then followed the derivation rules (for powers, also with fractional exponents, for sums and differences, products, quotients and composition of functions).

Chapter 2 explained how to determine a tangent to a curve (namely as that side of the polygon given above which lies at the point under consideration). The determination of maxima and minima followed in the 3rd chapter. The 4th chapter began with the introduction of the 2nd derivative and dealt with turning points and peaks (singular points). In the following four chapters – supported by numerous illustrations – curvature and the circle of curvature, the evolute (= the path of the centre of the circle of curvature), enveloping curves and the like were examined.

Finally, in the last chapter, under the simple title of *Solving Some Problems that Depend on the Methods Previously Discussed*, came, among other things, the rule that today bears L'HÔPITAL's name.

He explained it by means of two examples:



Given the set of functions with $f_a(x) = \frac{\sqrt{2a^3x - x^4} - a \cdot \sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$ call the numerator and denominator functions n and d and then for the point $x = a$ they both take the value 0.

For the derivatives we get $n'(x) = \frac{a^3 - 2x^3}{\sqrt{2a^3x - x^4}} - \frac{a^3}{3 \cdot \sqrt[3]{a^2x}}$ with $n'(a) = -\frac{4}{3}a$ and $d'(x) = -\frac{3a}{4\sqrt[4]{a^3x}}$ with $d'(a) = -\frac{3}{4}$ and from this $\lim_{x \to a} f_a(x) = \frac{16}{9}a$.

Analogously, he showed that if the undefined value in the graph of the function $y = \frac{a^2 - ax}{a - \sqrt{ax}}$ at the point $x = a$ is given by $y = 2a$ then the function is continuous.

After the success of his book on differential calculus, L'HÔPITAL also planned a volume on integral calculus, but abandoned the idea when he learned that LEIBNIZ was preparing a book on the subject. Before his death, L'HÔPITAL was able to complete work on another work (*Traité analytique des sections coniques* – Analytical Treatment of Conic Sections), which was published posthumously.

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https://www.spektrum.de/wissen/guillaume-de-lhopital-1661-1704/1389714

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