GIROLAMO CARDANO & Niccolò Tartaglia

by HEINZ KLAUS STRICK, Germany

CARDANO's name is associated with one of the most violent disputes in the history of mathematics. Does the *CARDAN formula* really come from CARDANO himself? Or did he use the intellectual property of TARTAGLIA?

GIROLAMO (also known as GERONIMO or HIERONIMO) CARDANO was born as the illegitimate child of a lawyer in Milan. His father was a lecturer at the University of Pavia; he also taught geometry at the prestigious *Scuole Piatti* in Milan. (drawings: © Andreas Strick)

Taught mathematics by his father, the adolescent planned an academic career, but at the age of 19 decided to study medicine, which he began in Pavia and – because of the war between the Habsburgs and France for supremacy in northern Italy - continued in Padua.

After his father's death, the highly gifted student quickly squandered his inherited fortune; he financed his further studies by playing cards and dice. Among other things, he was concerned with the chances of obtaining a certain total with double and triple dice, if the dice had already been rolled once. From 1524 onwards, he summarised these first findings (together with advice on tactical behaviour) in a book: *Liber de Ludo Aleae* (*On Games of Chance*) which contained approaches like LAPLACE's concept of probability as well as the correct calculation rules for the addition and multiplication of probabilities. However, CARDANO was not interested in publishing it – he wanted to keep his advantage as long as possible.

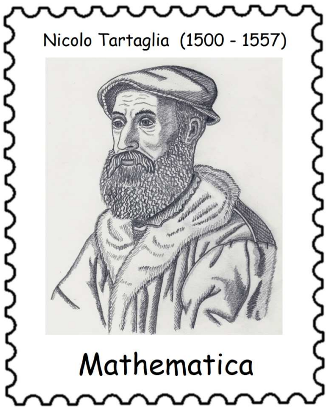
He successfully applied for the position of Rector of the University of Padua and obtained a doctorate in medicine. His attempt to establish himself as a doctor in Milan was refused by the local doctors, citing his illegitimate origin. However, this was only a pretext: they feared above all conflicts with the headstrong, uncompromising and often aggressive person.

So CARDANO settled down as a doctor in a village near Padua, but could hardly feed his young family from the income. After a run of bad luck in gambling, he had to sell his wife's furniture and jewelery and moved temporarily into the poorhouse. He was happy when he was offered a job as a mathematics teacher at the *Scuole Piatti*, where his father once taught. Now living in Milan, he secretly treated sick people there. He was so successful that even local doctors eventually asked him for advice. He forfeited his chances of being officially licensed as a doctor when he published a book in 1536 in which he questioned the qualifications and character of the Milanese doctors. In the meantime, however, the number of admirers of his abilities as a doctor had grown so great that he was finally granted admission to practise in Milan in 1539. The reason given was that his father later married his mother and thus his birth was subsequently legitimized.

In the same year CARDANO published two books on mathematics, including *Practica arithmetice et mensurandi singularis*, in which he gave a different, but also incorrect solution to LUCA PACIOLI's problem of division:

- *How is the stake of two players to be divided fairly if a game has to be abandoned prematurely and cannot be continued*?

The year 1539 was also the year Cardano made contact with Tartaglia ...



Nicolo Fontana was the son of a mounted courier who carried mail from Brescia (in the Republic of Venice) to neighbouring towns. When he was the victim of a robbery, the family was impoverished and his widow had great difficulty in supporting her three children.

In 1512, French troops conquered the city and took revenge for a defeat suffered before. Over 40,000 inhabitants of the city were indiscriminately killed. Nicolo took refuge in the cathedral and did not escape the slaughter there either; he survived badly injured.

His face was disfigured by deep wounds; as an adult he tried to cover the scars with a thick beard. Since the terrible experience he stuttered and his neighbours called him Tartaglia, meaning stutterer – he took this name as his first name. Nicolo attracted attention because of his talent for mathematics, and his mother found someone willing to sponsor him. Soon he was giving mathematics lessons himself, first for entrance classes, and later in Verona and Venice for older pupils.

Tartaglia dealt with cubic equations for which no method of solution had been found until then. Pacioli had generally considered them to be unsolvable ("*impossibile*"). At the beginning of the 16th century, equations were still formulated in words, and since calculating with negative numbers was not yet developed, the coefficients were basically positive numbers. Therefore solution methods for 13 different types of equations had to be found.

There were seven equation types contain all powers of the unknown: $x^3 + ax^2 + bx + c = 0$; $x^3 + bx + c = ax^2$; $x^3 + ax^2 + c = bx$; $x^3 + ax^2 + bx = c$; $x^3 + c = ax^2 + bx$; $x^3 + bx = ax^2 + c$; $x^3 + ax^2 = bx + c$; and there were another six types with some of the powers missing: $x^3 + bx + c = 0$; $x^3 + c = bx$; $x^3 + bx = c$ and $x^3 + ax^2 + c = 0$; $x^3 + c = ax^2$; $x^3 + ax^2 = c$.

Scipione dal Ferro (1465 – 1526), lecturer at the University of Bologna, succeeded around 1515 in developing a process that led to a solution in the case of $x^3 + bx = c$. He did not publish this, however, but entrusted the procedure to one of his students, Antonio Fior, among others. In order to attract public attention, Fior spread the word that he was able to solve cubic equations and – as was customary at the time – challenged other mathematicians to a public competition.

At the beginning of 1535 Tartaglia accepted the challenge, because in the meantime he had worked out how to solve equations of the type $x^3 + ax^2 = c$. Each of the contestants set thirty tasks for his opponent to solve within a period of forty days. While Tartaglia posed problems with varying degrees of difficulty, all of Fior's tasks are of type $x^3 + bx = c$.

For example, the first problem is:

- *Find me a number such that when its cube is added, the result is 6.* (So here is the equation to solve is $x^3 + x = 6$).

Only on the last night before the deadline Tartaglia found how to solve such equations.

Fior, on the other hand, proved to be a mediocre mathematician who relied on his opponent not being able to solve this type of cubic equation.

Generously, Tartaglia gave up the prize offered for the competition; what was important to him was fame, which he hoped will lead to an occupation that was more secure and prestigious than his previous one.

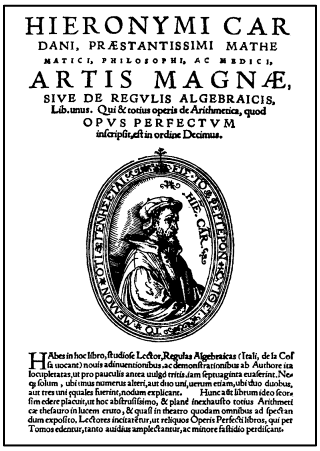
CARDANO, who until then had accepted PACIOLI's opinion that cubic equations could not be solved, heard of the contest. Through a bookseller he asked TARTAGLIA if he could provide the method he had found for a book on algebra which was in preparation. TARTAGLIA told him that he would publish the methods he had found in a book of his own.

CARDANO now personally urged him to communicate the method, suggesting that he could use his good contacts with the Governor of Milan to finally find him a suitable job. TARTAGLIA was prepared to reveal his secret, but made CARDANO swear that he would keep the method to himself and not publish it. On his journey home, TARTAGLIA regretted what he had done, but was reassured when his method did not appear in any of CARDANO's mathematics books published in the same year.

TARTAGLIA told CARDANO the method he found on the decisive night in verse form.

According to this, one finds the solution of an equation $x^3 + bx = c$ by the following steps:

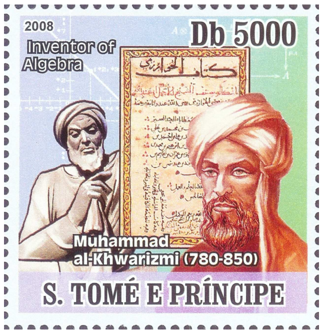
- Search for two numbers *u* and *v* whose difference is *c* and whose product is equal to the cube of $b/3$. Then $\sqrt[3]{u} - \sqrt[3]{v}$ is the solution sought.



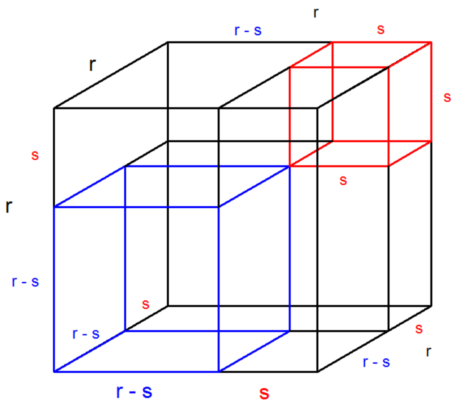
CARDANO was able to use the information from TARTAGLIA to develop methods to solve all types of cubic equations. His student LODOVICO FERRARI even succeeded in solving fourth-degree equations using a similar approach.

In 1543, the two learnt from DEL FERRO's son-in-law that DEL FERRO had already been able to solve the first type of cubic equation in 1515 and that TARTAGLIA was therefore not the first to be able to do so.

In 1545 CARDANO's major work appeared in Nuremberg: *Ars magna* (*Ars magnae sive de Regulis Algebraicis*). In 40 chapters, it shows how to solve the different types of third and fourth degree equations. Explicitly DEL FERRO and TARTAGLIA as well as FERRARI were mentioned as discoverers of the solutions of certain types of equations.



For the solution of the equation of type $x^3 + bx = c$ CARDANO used an identity which he proved by decomposing a cube in analogy to AL-KHWARIZMI's decomposition of squares:



$(r-s)^3 = r^3 - 3r^2s + 3rs^2 - s^3$, and so $r^3 - s^3 = (r-s)^3 + 3rs \cdot (r-s)$.

If one sets $b = 3rs$ and $c = r^3 - s^3$, then the equation becomes: $x^3 + 3rs \cdot x = r^3 - s^3$ for which $r - s$ is obviously a solution.

In order to solve the original cubic equation, one only has to determine suitable values for *r* and *s*.

From $s = \frac{b}{3r}$ it follows that $r^3 - \frac{b^3}{27r^3} = c$.

This results in $27r^6 - b^3 = 27cr^3$, and also $r^6 - cr^3 = \frac{b^3}{27}$.

This is a quadratic equation in r^3 which can be solved by completing the square:
$(r^3 - \frac{c}{2})^2 = (\frac{b}{3})^3 + (\frac{c}{2})^2$, and so $r = \sqrt[3]{\frac{c}{2} + \sqrt{(\frac{b}{3})^3 + (\frac{c}{2})^2}}$.

Analogously one puts: $(s^3 + \frac{c}{2})^2 = (\frac{b}{3})^3 + (\frac{c}{2})^2$, and so $s = \sqrt[3]{-\frac{c}{2} + \sqrt{(\frac{b}{3})^3 + (\frac{c}{2})^2}}$.

Hence the solution $x = r - s = \sqrt[3]{\frac{c}{2} + \sqrt{(\frac{b}{3})^3 + (\frac{c}{2})^2}} - \sqrt[3]{-\frac{c}{2} + \sqrt{(\frac{b}{3})^3 + (\frac{c}{2})^2}}$.

Example: The equation $x^3 + 6x = 2$, so with b = 6 and c = 2, we therefore get the number

$$x = \sqrt[3]{\frac{2}{2} + \sqrt{(\frac{6}{3})^3 + (\frac{2}{2})^2}} - \sqrt[3]{-\frac{2}{2} + \sqrt{(\frac{6}{3})^3 + (\frac{2}{2})^2}} = \sqrt[3]{1 + \sqrt{9}} - \sqrt[3]{-1 + \sqrt{9}} = \sqrt[3]{4} - \sqrt[3]{2}$$

as a solution.

Of course, the *Ars Magna* did not yet contain transformations as they are printed here; rather, Cardano gave instructions on how to calculate from the coefficients b and c and gradually built up the terms of the solution formula.

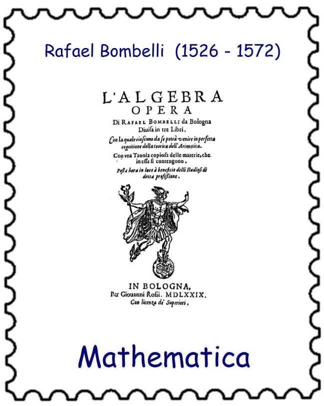
Surprisingly, knowledge of the solution method for equations of the type $x^3 + bx = c$ is sufficient, because by a suitable substitution the general equation $x^3 + ax^2 + bx + c = 0$ can be reduced to an equation of this form. If one replaces *x* by $y - \frac{a}{3}$, then $y^3 + (b - \frac{a^2}{3}) \cdot y = -\frac{2a^3}{27} + \frac{ab}{3} - c$, and is thus an equation of the type $y^3 + m \cdot y = n$.

In studying the various cases CARDANO also encounters the problem of roots of negative numbers: For example:

- *Is it possible to divide the number 10 into two summands in such a way that their product is 40?*

The solution to the problem, namely the decomposition of 10 into the summands $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$, he describes as both *sophisticated* and *useless*.

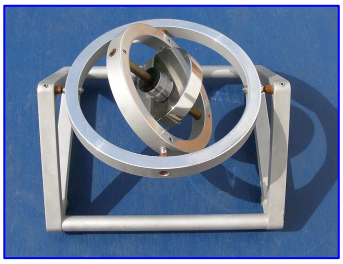
Only later was RAFAEL BOMBELLI (1526 – 1572) able to deal with imaginary quantities and formulate the corresponding calculation laws.



TARTAGLIA was furious when he learned of CARDANO's publication and accused him of perjury. He did not react; he left the public debate to FERRARI. In 1548 TARTAGLIA refused to enter into a competition with FERRARI until he was promised a prominent position in Brescia in the event of a victory. At the end of the first day of the competition, TARTAGLIA realised that FERRARI was superior to him and he left the place of the competition. In Brescia, he lectured on EUCLID for a year before he was told that he would not be paid for it, as he was inferior to FERRARI.

TARTAGLIA spent the last years of his life as a teacher in Venice; in 1557 he died embittered and impoverished. Even though he was surpassed by CARDANO in solving cubic equations, his life's work is not insignificant: in 1537 he published the work *Nova Scientia*, which dealt with ballistic issues (it contains, among other things, the observation that the greatest firing range is achieved at a firing angle of 45°). TARTAGLIA was the first to translate EUCLID's *Elements* into Italian and published a translation of ARCHIMEDES' works.

CARDANO, on the other hand, after the publication of the *Ars Magna*, enjoyed his fame as the most important mathematician of his time. However, his success was not limited to mathematics: he published more than 200 works on medical, mathematical, physical and philosophical subjects. He was the first medical doctor to describe the symptoms of typhoid fever and to highlight the differences between syphilis and gonorrhoea. He received offers from various European dynasties, but only once left Italy: in Scotland he cured the Archbishop of St Andrews, who suffers from life-threatening asthma and he was paid a fortune by the Archbishop.



He proved his reputation as a universal scholar, among other things, by describing a technical principle that bears his name today: a *CARDANic suspension* makes it possible to rotate measuring instruments by means of two axes perpendicular to each other; this principle is also taken into account in the *CARDANic joint*.

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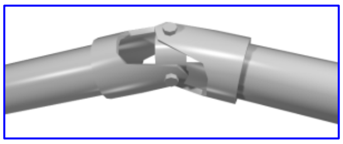

With the events of 1560, CARDANO's life changed dramatically: The marriage of his eldest son fell apart; in the end, the son poisoned his wife and was sentenced to death by the court. As the father of a murderer, CARDANO lost his reputation and employment. He accepted a professorship of medicine in Bologna, but quickly made enemies again through his arrogant behaviour. His second son gambled away everything he owned and, when his father stopped giving him money, he broke into his house to rob him. After the father reported the theft his son was banished from the city.

Then in 1570 CARDANO himself was imprisoned – for heresy: he had drawn up a horoscope of JESUS CHRIST. The Inquisition wanted to make a deterrent example of him; but the Pope, who valued him as a medical advisor, forgave him and even granted him an annuity. He lived in Rome until the end of his life and wrote his memoirs.

LEIBNIZ writes about CARDANO: "Despite all his faults, he was a great man; but without them he would have been incomparable".

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https://www.spektrum.de/wissen/girolamo-cardano-1501-1576-und-nicolo-tartaglia-1500-1557/1160795

Translated 2020 by John O’Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps; Enquiries at europablocks@web.de with the note: "Mathstamps".