The Kampyle of Eudoxus,
Or Is It?

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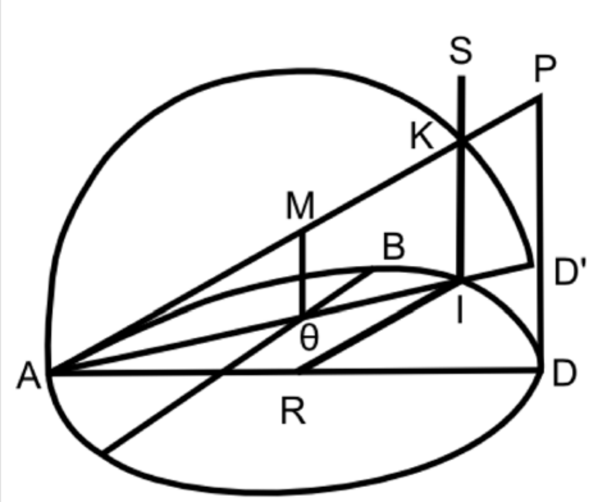
In a third century letter written to Ptolemy, Eratosthenes described the curves Eudoxus used to solve the classical problem of cube duplication as "... that shape which is curved in the lines that divine Eudoxus constructed." "That shape," or, at least one he thought to be it, was given a name by Paul Tannery[1] in the late 18th century when he conjectured that the "curved lines" in Eratosthenes' letter had a linguistic connection to "the curve" of Homer's bow. Tannery had found a curve he felt fit this description and called the curve a kampyle meaning simply "the curve" of Eudoxus.

Tannery, writing some 250 years after Descartes[2], made no mention that the curve Descartes' drew with his XYZ compass to solve the cube problem had this same "curve of the bow" shape. In 1989 Wilbur Knorr[3] proposed a rotational model of Archytas' solution of the cube problem that generates a "phantom" kampyle to find the two required means. Even Knorr seems not to have recognized his device was based on "Homer's bow" before his untimely death.

The simplicity of Knorr's model can be seen in this figure found in his book. It consists of, a base circle that has a diameter AD equal to the length of the longer given line and twice the length of the shorter line AB fitted into the circle as shown; a semicircle with diameter AD′=AD constructed perpendicular to the base so that it rotates about A; a bar RIS which is bent so that IS is perpendicular to the base and lies on the plane of the semicircle as it rotates about the center of the base R; and, a right triangle ADP whose hypotenuse lies on the plane with the semicircle and is marked so that AM=AB. The triangle can be rotated about its arm AD at the fixed angle DAM=DAB.



Knorr Figure 5.10b

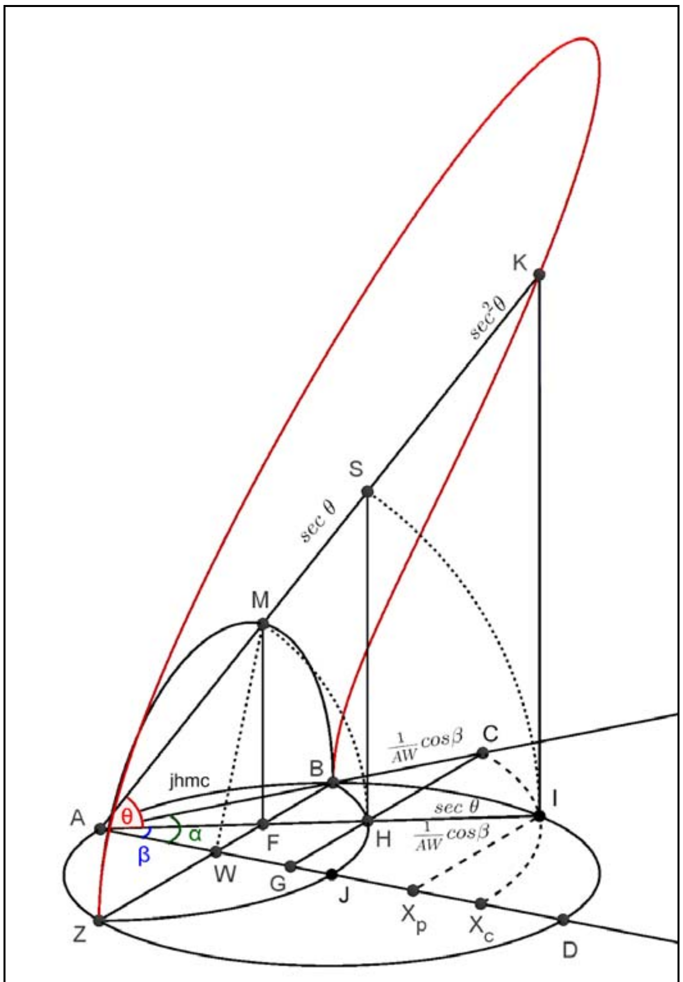
With points D′ and I initially concurrent with D, rotating AP counterclockwise about AD rotates the plane of the semicircle about A moving I along arc DB of the base circle and sliding AP across the face of the semicircle. The combined rotations cause point K, the intersection of AP with IS, to move along a path on the rotating plane described by $IK^2 = AI^4 - AI^2$ which is the equation of a Kampyle. The rotation is stopped when K becomes concurrent with the semicircle to mark the point where the kampyle intersects the semicircle. AK and AI, the two means between AD′ and AM can then be found from the included triangle AKD′.

The "surfaces" interpretation of Archytas' solution[4] uses these same components but treats the rotating components as surface generators and manipulates them independently. After the base circle is drawn, IS is rotated around it to create an upright half cylinder on arc ABD. AP, initially lying along AB, is then rotated clockwise to create the cone surface and mark its intersections with the generators of the cylinder wall. The locus of these points above arc BD is the double curvature, cone-cylinder intersection curve. Then, ignoring the surfaces, the semicircle is rotated to intersect the just generated curve. This marks the same point on the semicircle as marked by Knorr's device and yields the same two means.

Tannery showed that the cone-cylinder intersection projects to the xz plane as a hyperbola of the form $z^2 = a^2x^2 - ax$. It will be shown that when the points of the rotating kampyle of Knorr's device are circularly mapped to the space above the base circle their 3D locus has this same projection to the xz plane to confirm that it is the cone-cylinder intersection curve.

The next figure elaborates on the mechanism underlying Knorr's device. It is to be noted that it is the diameter of the semicircle, not shown here, that sets the upper extreme. The diameter of the base circle is somewhat arbitrary so long as the shorter line is fitted into the circle as the chord AB to fix $\angle JAB$ and, in turn, the angle of rotation α, so that $sec(\alpha) = 1/AW = AD$.

The red curve is the 3D locus of point K (the intersection of AP with IS) when AP is rotated around AD generating a phantom kampyle on the rotating plane. When point $K(AI, IK)$ is circularly mapped to the xz plane using an arc of diameter AI, it maps to a point $K_c(x_c, z)$ where $x_c = AI$ and $z = IK$. AIK is a right triangle and $IK^2 = AI^4 - AI^2$. From this the curve of the points mapped from the red curve to the xz plane is the kampyle $z^2 = x_c^4 - x_c^2$. Or, conversely, the red curve is a circularly mapped plane kampyle. To show that it is also a cone-cylinder intersection curve it will be projected to the xz plane.



Kample generated cone-cylinder intersection

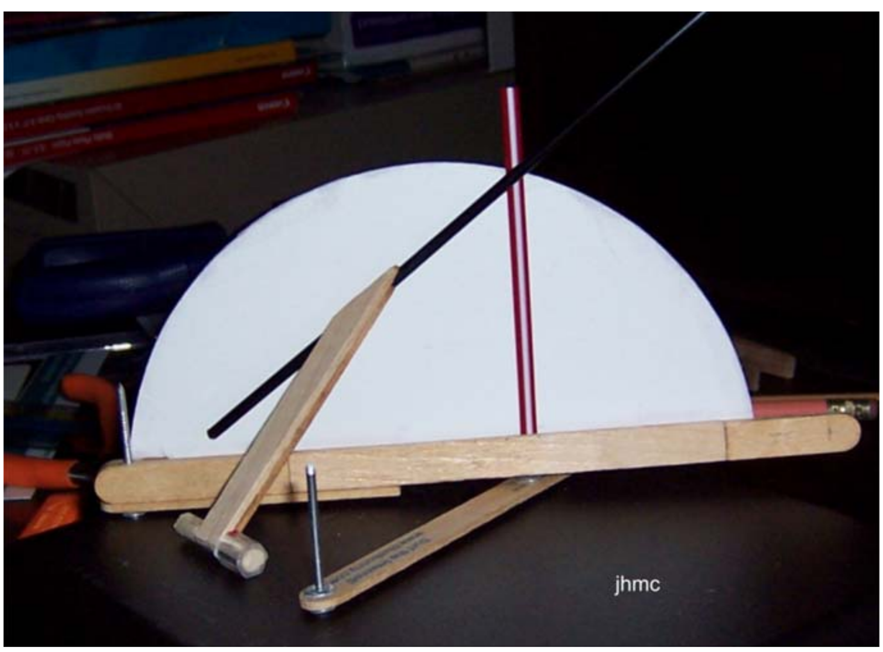
$AC/AB = AG/AW$, but for Archytas' device $AB = 1$ and $AW = 1/2$ making $AC = 2AG$.

C and S lie on the same circle of the cone making $AC = AS = AI$. Hence $AI = 2AG$.

$AI/x_p = AH/AG$ and since $AH = 1$, $x_p = AG * AI = AI^2/2$ while $z_p = IK$.

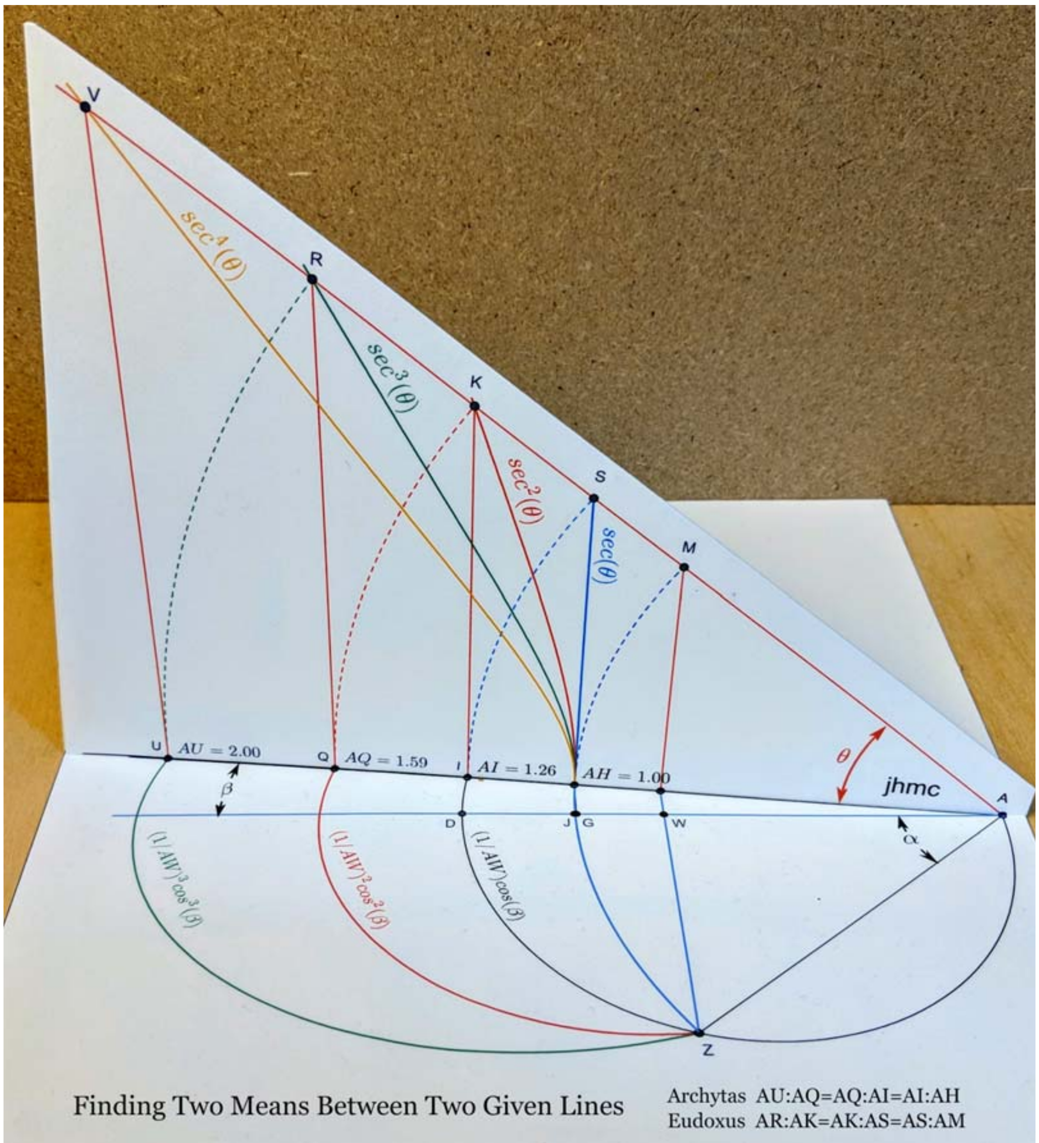
$IK^2 = AI^4 - AI^2$ and $z_p^2 = 4x_p^2 - 2x_p$

the same as Tannery's projection and confirms the cone-cylinder intersection curve is a circularly mapped kampyle.



A simple "proof of concept" implementation of Knorr's contrivance is shown in the picture to the left. It has been rotated so that the intersection of the black rotating bar with the red upright bar lies on the semicircle.

The “accepted” version of Archytas’ solution above conveniently ignores Eratosthenes assertion that Archytas used a difficult to construct figure of cylinder sections. This conundrum is easily resolved by a conjecture that the above solution was not Archytas’ first solution, but rather was one that combined the curved secant lines of an Eudoxian solution with their complementary cosine cylinder sections of an earlier solution as suggested in this popup picture.



Replacing some curves with a semicircle and removing others of the plane solutions that became redundant or unneeded was an easy morph to the simple 3D solution suggested by Knorr that generates no surfaces and draws no curves. It just works!

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References

1. Paul Tannery, “On the solutions of Delos problem by Archytas and Eudoxus.” Bordeaux, Soc. Sci. Mém., 2, 1878, pp. 277-283.

2. Paul J. Olscamp, “Rene Descartes, Discourse on Method, Optics, Geometry and Meteorology.” Geometry Third Book, pgs. 228-229. Hackett Publishing Co.

3. Wilbur Knorr, “Textual Studies in Ancient and Medieval Geometry,” pg. 109.

4. Marshall Clagett, “Archimedes in the Middle Ages Vol. I,” Bana Masu - Verba Filiorum, Proposition XVI - XVII, pages 336-343.