§32.1. ***Synopsis: Chapter Thirty Two.***

This final chapter provides a listing of the lengths associated with the 5 Platonic solids inscribed in a unit sphere, together with their logarithms. A few problems are then presented related to an octahedron of volume 17 units: these are solved by comparing the ratios of lengths, areas, etc., between shapes

§32.2. *Chapter Thirty Two.* [p.86.]

Concerning the Sphere, and the five Regular bodies inscribed in the same; the Tetrahedron, the Octahedron, the Cube, the Icosahedron, and the Dodecahedron.

We have shown the use of logarithms with plane figures in the above chapters; in this chapter we demonstrate the same use for the regular solid shapes also.

These five regular bodies are put together from equal pyramids with equal altitudes: the bases of which are apparent from the outside, but with the vertices meeting within at the centre [of the sphere]: the altitudes of which are equal to the perpendicular from the centre of the body to the centre of the base, or the same as the radius of the sphere inscribed in the body.

If the altitude of the pyramid is taken by a third of the area of the base, the product is the volume of the same. And therefore, the product of the radius of the inscribed sphere, by a third of the area of the surface of that regular body, is the volume of the same body.

If these five bodies are inscribed in the same sphere, the same circle circumscribes the [base] triangle of the icosahedron & the pentagon of the dodecahedron; likewise for the triangle of the octahedron, and the square of the cube.

For: if the faces of one solid are equal in number to the vertices of the other; [then] the same circle is circumscribed to the faces of both[1]: & the volumes of the bodies themselves are in proportion with these surface areas [as they have the same in-radius]. And the area of the Tetrahedron is to the area of the Cube, as the side of the equilateral triangle is to the diameter of the circle circumscribed [to the square][2]. But the [ratio of the] area of the icosahedron to the area of the Dodecahedron, as the side of the icosahedron to the side of the cube of the same inscribed sphere: or as the side of the equilateral triangle subtended to two sides of the pentagon .

.87.

			Logarithms
AR	For a sphere of radius one: there will be	- - - - - - - - - -	0000000000
A.	Periphery of largest circle	628318530718	0798179869
B.	Area of largest circle	3141592653589	0497149873
C.	Surface area of sphere	1256637061316	1099209864
D.	Volume of sphere	418879020439	0622088610

If the regular solids are inscribed in this sphere: there is:

T	*E.*	Side, _. $^{8}/_{3}$	16329931618	0212984366
e	*F.*	Base area, _. $^{4}/_{3}$	11547005384	0025576261
t	*G.*	Radius of circle circumscribing triangle, _.$^{8}/_{9}$	9428090416	– 0025576261
r	*H.*	Surface area, _. $^{64}/_{3}$	46188021536	0664529350
a'	*I.*	Radius of inscribed sphere, $^{1}/_{3}$	33333333333	– 0477121155
n	*K.*	Volume, _. $^{64}/_{243}$	5132002393	– 0289713150
O	*L.*	Side, _. 2	14142135624	0150514998
c	*M.*	Base area, _.$^{3}/_{4}$	866025404	– 0062469368
t	*MS.*	Radius of circle circumscribing triangle, _.$^{2}/_{3}$	8164965809	– 0088045629
a'	*N.*	Surface area, _. *48*	6928203230	0840620619
n	*O.*	Radius of inscribed sphere, _. $^{1}/_{3}$	5773502692	– 0238560627
	P.	Volume, $1^{1}/_{3}$	13333333333	– 0124938737
C	*Q.*	Side, _. $^{4}/_{3}$	11547005384	0062469368
u	*R.*	Base area, $^{4}/_{3}$	13333333333	0124938737
b	*S.*	Radius of circle circumscribing square, _.$^{2}/_{3}$	8164965809	– 0088045629
e	*T.*	Surface, area *8*	8 - - - - - - -	0903089987
	V.	Radius of inscribed sphere, _. $^{1}/_{3}$	5773502692	– 0238560627
	X.	Volume, _. $^{64}/_{27}$	15396007179	0187408105
I	*AA.*	Side, _. bin 2 – _.$^{4}/_{5}$	10514622242	0021793674
c	*AB*	Radius of circle circumscribing triangle, . bin _.$^{2}/_{3}$ – _.$^{4}/_{45}$	6070619981	– 0216766954
o	*AC*	Radius of inscribed sphere, _. bin $^{1}/_{3}$ + _.$^{4}/_{45}$	7946544723	– 0099821668
s	*AD*	Area of triangle, _. bin $^{9}/_{10}$ – _. $^{9}/_{20}$	4787270692	– 0311912015
a'	*AE.*	Area of surface, _. bin 360 – _. 72000	9574541383	0981117081
n	*AF.*	Volume, _. bin $^{49}/_{9}$ + _. $^{14100}/_{3645}$ [_. bin $^{40}/_{9}$ + _. $^{1600}/_{405}$]	2536150710	0404175058
D	*BA.*	Side, _. bin 2 – _.$^{20}/_{9}$	7136441796	– 0146518272
d	*BB.*	Radius of circle circumscribing pentagon, _. bin $^{2}/_{3}$ – _.$^{4}/_{45}$	6070619981	– 0216766954
e	*CB.*	Area of pentagon, _. bin $^{25}/_{18}$ – _. $^{125}/_{324}$	8762185202	– 0057387571
c	*BD.*	Area of surface, _. bin 200 – _. 8000	10514622242	1021793674
a'	*BE.*	Radius of inscribed sphere, _. bin. $^{1}/_{3}$ + _.$^{4}/_{45}$	7946544723	– 0099821668
n	*BF.*	Volume, _. bin $^{40}/_{9}$ + _. $^{8000}/_{729}$	2785163863	0444850752

[Table 32-1]

I can show briefly by means of some examples the use logarithms presents for these bodies. Let the given volume of the octahedron be 17: and these quantities are sought: 1, the radius of the circumscribed sphere. 2, The Surface area of the tetrahedron; 3. The side of the cube; 4. The radius of the circle for the circumscribed triangle of the icosahedron; 5. The volume of the dodecahedron, for the same sphere, with the given octahedron, and the rest of the inscribed bodies.

1. The Radius of the inscribed sphere is required. But when there is no ratio [given] between the different magnitudes; the logarithms of the cube roots of the given volumes are taken: as lines [i.e. lengths of corresponding lines] are compared with lines. And the logarithms of these roots

themselves (with the third of course taken of those given), the lengths of the sides themselves need not be evaluated, will do all the work for us.

.88.]

		Logarithms
Prop-ort-ions	Cube root of the volume of the octahedron P	0041646246
	Radius of circumscribed sphere *AR*	0000000000
	Cube root of the volume of the given octahedron, 17	0410149640
	Radius of the required sphere	0368503394
2.	The area of the tetrahedron is sought. Here the logarithms of the squares are found, in order that the comparison of the surfaces is established.	
Prop-ort-ions	Square of the cube root of the volume of the octahedron P	0083292492
	Area of the tetrahedron H	0664529360
	Square of the cube root of the given volume 17	0820299280
	Sum of the means	1484828640
	Surface of the required tetrahedron $25\underline{2078698}$	0401536248
3.	The side of the cube is sought	
Prop-ort-ions	Cube root of the volume of the octahedron *P*, Complementary Arith.	9958353754
	Side of the cube *Q*	0062469368
	Cube root of the given volume 17	0410149640
	Side of the cube required $2\underline{60757024}$	10430972762
4.	The radius of the circle of the circumscribed triangle of the icosahedron is required	
	Cube root of the volume of the octahedron P	0041646246
	Radius of the circle of the circumscribed triangle AB	– 0216766954
	Cube root of the given volume 17	0410149640
	Sum of means	0193382686
	required radius of circle circumscribing triangle of icosa. $1\underline{418196607}$	0151736440
5.	Volume of dodecahedron required.	
	Cube root of the volume of the octahedron *P*, Complementary Arith.	9958353754
	Volume of dodecahedron *BF*	0444850752
	Volume of the octahedron given 17	1230448921
	Volume of dodecahedron sought $35\underline{51083922}$	11550360936

[Table 32-2]

And in this way, I have thought to explain gradually some of the uses of logarithms, in this part of the work. Yet their most outstanding use still remains, and this is especially necessary in the teaching of Spherical Trigonometry: which separately, I hope, I will show in my own book[3]. God willing. To whom alone in all things is due all the glory.

FINIS.

§32.3. *Notes on Chapter Thirty Two*

[1] These pairs of regular figures are dual: that is, the vertices of one correspond to the faces of the other, and vice versa. Thus, the tetrahedron has 4 Faces and 4 Vertices, or (4F,4V) and is self dual; while the cube (6F, 8V) and the octahedron (8F,6V) are dual ; and similarly the dodecahedron (12F, 20V) and the icosahedron (20F,12V) are dual figures. The interested reader can find most of Briggs' results for this chapter in standard references such as the *CRC Concise Encyclopaedia of Mathematics*, by Eric Weisstein, Chapman & Hall. CRC (1998). It may well be the case that Briggs' interest was sparked or augmented from his reading of the *Pantometria* by Thomas Digges, a fascinating book written some years earlier, which built upon the work of the father Leonard Digges, (who appears to have invented the first telescopes - in conjunction with his surveying work). This work contains numerous theorems on the Platonic solids.

[2] For the area (or volume) tetrahedron : cube as $1 : \sqrt{3}$ (or 1 : 3). For the tetrahedron, the circum - radius r_T of the equilateral triangle may be used to generate the correct ratio using areas, which is $\sqrt{(8/9)}$ from Table 32-1, while the diameter d_C of the circum-circle for the square is $2\sqrt{(2/3)}$, and $r_T/d_C = 1/\sqrt{3}$; or by some other arrangement.

[3] Postscript: Briggs' prayer was answered only in part. The present work was published in 1624. The companion work he refers to, which came to be called the *Trigonometria Britannica*, was published posthumously in 1633. Briggs had completed Part I of this work, dealing with the composition of the tables, before his death in 1631. Subsequently, his friend and colleague, Henry Gellibrand, Professor of Astronomy at Gresham College, completed the work by demonstrating the use of the logarithms in both plane and spherical trigonometry in Part II.

A Final Postscript:

As we have seen, Briggs Tables of Logarithms were all his own work: one has to marvel at his dedication in the almost endless pursuit of numbers to achieve his goal. The hiatus between 20,000 and 90,000 surely indicates that he had reached his limit, for his health was breaking down: he knew how to calculate the logarithms of the missing numbers, but did not wish to spend the rest of his days doing so. Instead, he diverted his attention to trigonometric tables, which as we have noted he did not complete either. Thus, the actual tables were left in a state of limbo: even those printed were riddled with printer's errors. A.J. Thompson in his *Logarithmica Britannica* gives a list of these errors, which amounts to some 1200 in 30,000 numbers, or 4%. One has to conclude that the proof-reading was inadequate, though this in itself was an arduous task: to check 30,000 numbers, each of 14 digits, meant checking some 400,000 individual digits.

The work of Vlacq and his associates, who treated the task as a business proposition, finished the tables off easily by reducing the number of places so that the first interpolation scheme could still be used, and so took the work from the gentle shades of academia into the hustle and bustle of the everyday life of navigation, commerce, engineering, surveying, etc. However, errors were to continue to plague mathematical tables for at least another 150 years, until Charles Hutton took matters in hand to produce error-free tables.

§32.4. Caput XXXII. [p.86.]

De Sphaera, et quinque corporibus Ordinatis eidem inscriptis; Tetraehedro, Octaehedra, Cubo, Icosaeedro, Dodecaedro.

Logarithmorum usum in figuris planis, superioribus capitibus ostendimus: eundem etiam, hoc capite, in figures solidis ostendemus. Haec quinque corpora Ordinata componuntur e pyramidibus, aequalibus, et aequaealtis: quarum bases extra apparent, vertices autem intus in centro concurrunt. Harum altitudines aequantur perpendicularibus a centro corporis, in centrum basis; vel radio Sphaerae eidem corpori inscriptae.

Si altitudo pyramidis ducatur in trientem basis, factus erit soliditas eiusdem. et idcirco factus a radio Sphaerae inscriptae, in trientem superficie corporis cuiusuis ordinati, aequabitur solidati eiusdem corporis.

Si haec quinque corpora eidem sphaerae inscribantur, idem circulus circumscribetur Triangulo Icosaedri, et Quinquangelo Dodecaedri: item Triangulo Octaedri, et Cubi Quadrato.

Nam; si hedrae unius, numero aequentur solidius angulis alterius; idem circulus circumscribetur hedris utriusque: et ipsa corpora sunt suis superficiebus proportionalia.

Estque Tetraedrum ad Cubum, ut latus trianguli aequilateri ad diametrum circuli circumscripti. Icosaedrum autem ad Dodecaedrum; ut latus Icosaedri ad latus cubi eidem sphaerae inscripti: vel ut latus Trianguli aequilateri ad subtendentem duo latera Quinquanguli.

[p.87.]

			Logarithmi
AR.	Si Radius Sphaerae sit unitas. erit	- - - - - - - - - -	0000000000
A.	Peripheria circuli maximi	6<u>28318530718</u>	0798179869
B.	Area maximi circuli	3<u>141592653589</u>	0497149873
C.	Superficies Sphaerae	12<u>56637061316</u>	1099209864
D.	Soliditas Sphaerae	4<u>18879020439</u>	0622088610

Si huic Sphaerae corpora Ordinata inscribantur: erit

E.	Tetraedri	Latus, _. 8/3	16329931618		0212984366
F.		Basis, _. 4/3	11547005384		0025576261
G.		Radius circuli triangulo circumscripti, _.8/9	9428090416	–	0025576261
H.		Superficies, _. 64/3	46188021536		0664529350
I.		Radius Sphaerae inscriptae , 1/3	3333333333	–	0477121155
K.		Soliditas, _. 64/243	5132002393	–	0289713150
L.	Octaedri	Latus, _. 2	14142135624		0150514998
M.		Basis, _.3/4	866025404	–	0062469368
MS.		Radius circuli triangulo circumscripti, _.2/3	8164965809	–	0088045629
N.		Superficies, _. *48*	6928203230		0840620619
O.		Radius Sphaerae inscriptae, _. 1/3	5773502692	–	0238560627
P.		Soliditas, 1 1/3	13333333333	–	0124938737
Q.	Cubi	Latus, _. 4/3	11547005384		0062469368
R.		Basis, 4/3	13333333333		0124938737
S.		Radius circuli quadrato circumscripti, _.2/3	8164965809	–	0088045629
T.		Superficies, area *8*	8 - - - - - - -		0903089987
V.		Radius Sphaerae inscriptae, _. 1/3	5773502692	–	0238560627
X.		Soliditas, _. 64/27	15396007179		0187408105
AA.	*Icosaedri*	Latus, _. bin 2 – _.4/5	10514622242		0021793674
AB		Radius circuli triangulo circumscripti, bin _.2/3 – _.4/45	6070619981	–	0216766954
AC		Radius Sphaerae inscriptae, _. bin 1/3 + _.4/45	7946544723	–	0099821668
AD		Area Trianguli, _. bin 9/10 – _. 9/20	4787270692	–	0311912015
AE.		Superficies, _. bin 360 – _. 72000	9574541383		0981117081
AF.		Soliditas, _. bin 49/9 + _. 14100/3645 [_. bin 40/9 + _. 1600/405]	2536150710		0404175058
BA.	Dodecaedri	Latus, _. bin 2 – _.20/9	7136441796	–	0146518272
BB.		Radius circuli Quinq. circumscripti, _. bin 2/3 – _.4/45	6070619981	–	0216766954
CB.		Area Quinquanguli, _. bin 25/18 – _. 125/324	8762185202	–	0057387571
BD.		Superficies, _. bin 200 – _. 8000	10514622242		1021793674
BE.		Radius Sphaerae inscriptae, _. bin. 1/3 + _.4/45	7946544723	–	0099821668
BF.		Soliditas, _. bin 40/9 + _. 8000/729	2785163863		0444850752

Quem usum in his corporibus hi praebeant Logarithmi, aliquot exemplis quam potero brevissime ostendam. Esto data soliditas Octaedri 17: et quaerantur: 1, Radius Sphaerae eidem circumscriptae ; 2, Superficies Tetraedri. 3, latus Cubi. 4, Radius circuli Icosaedri triangulo circumscripti. 5, soliditas Dodecaedri, eidem Sphaerae, cum dato Octaedro, reliquisque corporibus inscripti.

1. Quaeritur Radius Sphaerae circumscriptae. Cum autem nulla sit ratio inter magnitudines heterogeneas; sumendi sunt Logarithmi laterum cubicorum soliditatum datarum: ut lineae cum lineis conferantur. Ipsique Logarithmi laterum (triente scilicet datorum) licet ipsa latera ignorentur; totum nobis negotium conficient.

		Logarithms
pro-port .	Latus cubicum Soliditatis Octaedri P	0041646246
	Radius Sphaerae circumscriptae *AR*	0000000000
	Latus cubicum Soliditatis Octaedri dati 17	0410149640
	Radius Sphaerae quaesitus 233616436	0368503394

2. Quaeritur superficies Tetraedri .Hic quadratorium Logarithmi sumendi sunt, ut superficierum comparatio instituatur.

pro-port.	Quadratum lateris cubici Soliditatis Octaedri P	0083292492
	Superficies Tetraedri H	0664529360
	Quadratum lateris cubici Soliditatis datae17	0820299280
	aggregatum mediorum	1484828640
	Superficies Tetraedri quaesita 252078698	0401536248

3.	Quaeritur Latus Cubi	
pro-port.	Latus cubicum soliditatis Octaedri *P*, Complem. Arith.	9958353754
	Latus Cubi *Q*	0062469368
	Latus Cubi quaesitum datae 17	0410149640
	Latus Cubi quaesitum 260757024	10430972762

4. Quaeritur Radius circuli circumscripti triangulo Icosaedri.

pro-port.	Latus cubicum soliditatis Octaedri P	0041646246
	Radius circuli triangulo circumscripti AB	– 0216766954
	Latus cubicum soliditatis datae 17	0410149640
	aggregatum mediorum	0193382686
	Radius circuli circumscripti trianguli Icosa. 1418196607	0151736440

5. Quaeritur soliditas Dodecaedri.

pro-port.	Soliditas Octaedri *P*, Complem. Arith.	9958353754
	Soliditas Dodecaedri *BF*	0444850752
	Soliditas Octaedri data 17	1230448921
	Soliditas Dodecaedri quaesita 3551083922	11550360936

Atque ad hunc modum, usum Logarithmorum aliqua ex parte hoc tempore illustrandum censui. Superest adhuc eorum usus nobilissimus, et maxime necessarius in doctrina Triangulorum Sphaericorum: quem seorsum, uti spero, peculiari libro exhibebo. Volente DEO. Cui soli in omnibus omnis debetur gloria.

FINIS.