Often called al-Hāsīb al-Miṣrī ("the reckoner of Egypt") Abū Kāmil was one of Islam’s greatest algebraists in the period following the earliest Muslim algebraist, al-Khwārizmī (fl. ca. 825). In the Arab world this was a period of intellectual ferment, paric ularl in mathematics and the sciences.

There is virtually no biographical material available on Abū Kāmil. He is first mentioned by al-Nadīm in a bibliographical work, The Fihrist (987), where he listed with other mathematicians under “The New Reckoners and Arithmeticians,” which refers to those mathematicians who concerned themselves with the practical algorithms, citizens’ arithmetic, and practical geometry (see Bibliography). Ibn Khaldūn (1322–1406) state that Abū Kāmil wrote his algebra after the first such work by al-Khwārizmī, and Hajjī Khalīfa (1608–1658) attributed to him a work supposedly concerned with algebraic solutions of inheritance problems.

Among the works of Abū Kāmil extant in manuscripts is the Kitāb al-ṭarā‘if fī l-ḥiṣāb ("Book of Rare Things in the Art of Calculation"). According to H. Suter this text is concerned with integral solutions of some indeterminate equations; much earlier, Diophantus (ca. first century a.d.) had concerned himself with rational, not exclusively integral, solutions. Abū Kāmil’s solutions are found by an ordered and very systematic procedure. Although indeterminate equations with integral solutions had been well known in ancient Mesopotamia, it was not until about 1150 that they appeared well developed in India. Aryabhata (b. a.d. 476) had used continued fractions in solutions, but there is uncertain evidence that this knowledge had been passed on in any ordered form to the Arabs by the time of Abū Kāmil.

A work of both geometric and algebraic interest is the Kitāb … al-mukhārman mas wa’l-mu‘ashshar … ("On the Pentagon and Decagon"). The text is algebraic in treatment and contains solutions for a fourth-degree equation and for mixed quadratics with irrational coefficients. Much of the text was utilized by Leonardo Fibonacci (1175-ca. 1250) in his Practica geometriae. Some of the equations solved by Abū Kāmil in this work read as follows:

\[ x^4 + q = px \]

with \( x \) the side of a regular polygon inscribed in a circle. Also

\[ x^4 + q = px^2 \]

with \( x \) the side of a regular polygon (pentagon here) which has an inscribed circle.

The outstanding advance of Abū Kāmil over al-Khwārizmī, as seen from these equations, is in the use of irrational coefficients. Another manuscript, which is independent of the Tarā‘if, mentioned above, is the most advanced work on indeterminate equations by Abū Kāmil. The solutions are not restricted to integers; in fact, most are in rational form. Four of the more mathematically interesting problems are given below in modern notation. It must be remembered that Abū Kāmil gave all his problems rhetorically; in this text, his only mathematical notation was of integers.

Many of the problems in Kitāb fi'l-jabr wa'l-muqābala had been previously solved by al-Khwārizmī. In Abū Kāmil’s work, a solution for \( x^4 \) was worked out directly instead of first solving for \( x \). Euclid had taken account of the condition \( x < \sqrt{p/2} \) in \( x^2 + q = px \), whereas Abū Kāmil also solved the case of \( x > \sqrt{p/2} \) in this equation.

Abū Kāmil was the first Muslim to use powers greater than \( x^2 \) with ease. He used \( x^4 \) (called “square square square square”), \( x^6 \) (called “cube cube”), \( x^8 \) (called “square root”), and \( x^9 \) (called “cube”), as well as \( x^2 \) (called “square”). From this, it appears that Abū Kāmil’s nomenclature indicates that he added “exponents.” In the Indian nomenclature a “square cube” is \( x^6 \), in contradistinction. Diophantus (ca. a.d. 86) also added “powers,” but his work was probably unknown to the Arabs until Abu'l Wafā’ (940–998) translated his work into Arabic (ca. 998).

Abū Kāmil, following al-Khwārizmī, when using jadhr (“root”) as the side of a square, multiplied it by the square unit to get the area \( x^2 \). This method is older than al-Khwārizmī’s method and is to be found in the Mishnat ha-Middot, the oldest Hebrew geometry, which dates back to a.d. 150. This idea of root is related to the Egyptian khet (“cubit strip”).

The Babylonians stressed the algebraic form of geometry as did al-Khwārizmī. However, Abū Kāmil not only drew heavily on the latter but he also derived much from Heron of Alexandria and Euclid. Thus he was in a position to put together a sophisticated algebra with an elaborated geometry. In actuality, the resulting work was more abstract than al-Khwārizmī’s and more practical than Euclid’s. Thus Abū Kāmil effected the integration of ancient Mesopotamian practice and Greek theory to yield a wider approach to algebra.
It is possible that Greek algebra was known to Abū Kāmil through Heron of Alexandria, although a direct connection is difficult to prove. The influence of Heron is, however, definite in Abraham bar Hiyy’s work. That Abū Kāmil influenced both al-Karajī and Leonardo Fibonacci may be demonstrated from the examples they copied from his work. Thus through Abū Kāmil, mathematical abstraction, elaborated together with a more practical mathematical methodology, impelled the formal development of algebra.

NOTES

1. “Das Buch der Seltenheiten.”
3. Ibid., p. 37. Levey will soon publish the Arabic text of “On the Pentagon and Decagon,” discovered by him.
4. At least twenty of these problems from this text may be found in Leonardo Fibonacci, Scritti, Vol. 1, sect. 15: Vol. II.
8. Ibid., pp. 178, 184, 186, 202.

BIBLIOGRAPHY

1. Original Works. The following manuscripts of Abū Kāmil are available: Kitāb fi ‘l-jabr wa’l-muqābala (“Book on Algebra,” Paris BN MS Lat. 7377A; Munich Cod. MS Heb.225; Istanbul-MS Kara Mustafa 379), trans. into Hebrew by Mordecai Finzi ca. 1460. See also Kitāb al-ṭarā’if fi ‘l’sūsāb (“Book of Rare Things in the Art of Calculation,” Leiden, MS Arabic 1003, ff. 50r–58r; translations are found in Munich Cod. MS Heb. 225 and in Paris BN MS Lat. 7377A); Kitāb... al-mukhammas wa’l-mu’ashshar... (“On the Pentagon and Decagon,” Paris BN MS Lat.7377A; Munich Cod. MS Heb. 225; Istanbul-MS Kara Mustafa 379, ff.67r–75r); Al-waṣayā bi’l-judhār (MS Mosul 294) discusses the ordering of roots. Works of Abū Kāmil listed in The Fihrist of al-Nadim (p.281) include Kitāb al-falāḥ (“Book of Fortune”) Kitāb miftāḥ al-falāḥ (“Book of the Key to Fortune”). Kitāb fi ‘l-jabr wa’l-muqābala (“Book on Algebra”), Kitāb al-μiṣwāt wa’l-handasa (“Book on Surveying and Geometry”), Kitāb al-kifayā (“Book of the Adequate”), Kitāb al-ṭayr (“Book on Omens”), Kitāb al-asr (“Book of the Kernel”), Kitāb al-ḫaṭa’ā ʿayn (“Book of the Two Errors”), Kitāb al-jamʿ wa’l-tafriq (“Book on Augmentation and Diminution”).


Some of the more interesting literature to be found in the Algebra, in modern notation, are: