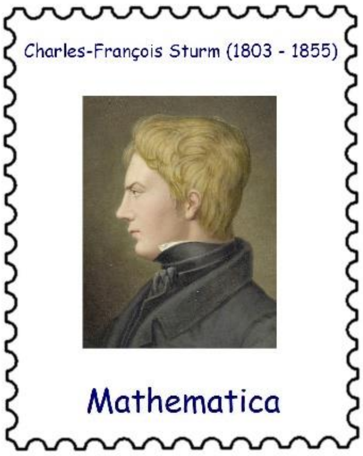
CHARLES-FRANÇOIS STURM (September 29, 1803 – December 18, 1855)

by HEINZ KLAUS STRICK, Germany



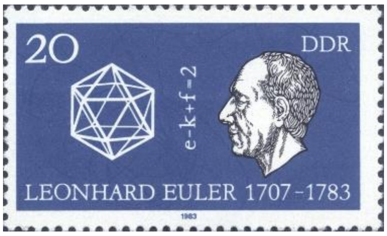
He is one of the 72 famous scientists and engineers whose name GUSTAV EIFFEL had inscribed in gold letters on the frieze of the first floor of the tower he built in Paris in 1889 – one hundred years after the French Revolution.

In 1836 the mathematician was elected to succeed ANDRÉ-MARIE AMPÈRE as a member of the *Académie des Sciences*. He was also a corresponding member of the *Prussian* and *Russian Academy of Sciences* (1835/36) and of the *Royal Society* (1840), which even awarded him the COPLEY Medal.

We are talking about JACQUES CHARLES FRANÇOIS STURM, who was born in Geneva in 1803, which after the annexation by French troops in 1798 became the capital of the newly founded *Léman* department. After the Congress of Vienna, the region around Geneva was admitted to the Swiss Confederation as the 22nd canton.

STURM's family originally came from the Strasbourg area. CHARLES FRANÇOIS's father supported the family by teaching arithmetic. During his school days, CHARLES FRANÇOIS showed more interest in Latin and Greek than in mathematics. In order to improve his knowledge of German, he regularly attended the services of the Lutheran congregation, as the sermons were held in German.

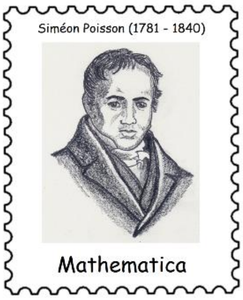
After leaving school, CHARLES FRANÇOIS's interests shifted from ancient languages to mathematics. SIMON L'HUILLIER, his first teacher at the *Académie de Genève* (latin: *Schola Genevensis*, today: *Université de Genève*) quickly recognized the special talent of the young student, advised and encouraged him, and even lent him his own books; because after the death of CHARLES FRANÇOIS's father, the family got into financial difficulties.



By the way: L'HUILLIER came up with the term "lim" used today in mathematics for the formation of limits for sequences and functions; he used them in a paper on the concept of infinity, for which he received a prize from the *Prussian Academy of Sciences* in 1786. In addition, L'HUILLIER worked on special cases and generalizations of EULER 's polyhedron formula.

After L'HUILLIER's retirement, his successor JEAN-JACQUES SCHAUB also looked after the gifted student, who passed his exams in 1823. For financial reasons, STURM then accepted a position as private tutor at the nearby *Château de Coppet*, which was owned by Duke VICTOR DE BROGLIE and his wife ALBERTINE DE STAËL-HOLSTEIN (the youngest daughter of Madame DE STAËL).

At first, STURM was dissatisfied with his living situation, but soon realized that his job gave him enough free time to pursue his own mathematical investigations. First articles on geometry appeared in the *Annales de mathématiques pures et appliquées*. When the ducal family moved to Paris for six months, he followed them. He got to know ALEXANDER VON HUMBOLDT through Duke DE BROGLIE, who introduced him to the discussion group around FRANÇOIS ARAGO – this is how he got in touch with e.g. PIERRE-SIMON LAPLACE, SIMÉON-DENIS POISSON, JOSEPH FOURIER, JOSEPH-LOUIS GAY-LUSSAC and ANDRÉ-MARIE AMPÈRE.



After returning from Paris, he initially continued his work as a private tutor, then decided to devote himself entirely to his own scientific research. Together with his college friend Daniel Colladon, he worked on solving a problem that the *Académie des Sciences* had announced as a competition:

- *What is the compressibility of water?*

In this context, the two first tried to determine the speed of sound in water by taking measurements in Lake Geneva. During the actual compression experiments, Colladon was seriously injured. In Paris they attended the lectures of Ampère, Gay-Lussac, Augustin-Louis Cauchy and Sylvestre Lacroix, to improve their theoretical knowledge. Thanks to the support of Arago, with whom Sturm could live, they were allowed to use the university's laboratories for their experiments. Nevertheless, neither competition entry nor that of other participants was considered worthy of an award. A year later (1827) – in the meantime Ampère employed the two as his assistants and the experiments went satisfactorily – they received the prize money of 3000 francs advertised by the *Académie*, which ensured their livelihood for a long time. The speed of sound they measured in the water of 1435 m/s differed only slightly from the value calculated using the Poisson formula: 1437.8 m/s.

In 1829, Charles François Sturm presented his famous contribution *Mémoire sur la résolution des équations numériques* (Notes on the numerical solution of equations) to the *Académie*, which we shall return to below.

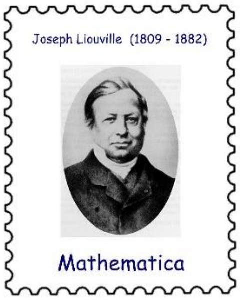
Sturm and Colladon tried in vain for permanent positions at state educational institutions. Despite the support of prominent members of the *Académie*, this was not successful because they were foreigners and also belonged to the Protestant denomination.

This only changed with the political changes after the July Revolution in 1830 when the Duke Victor de Broglie became the new Minister of Education.

STURM was employed as a professor for *mathématiques spéciales* at the *Collège Rollin* and COLLADON for mechanics at the *École Centrale des Arts et Manufactures*.

In 1833, STURM took French citizenship; he rejected an appointment made in the same year for a position at the *Académie de Genève*, as well as an offer from the University of Ghent.

His career was now on the up ...



After AMPÈRE's death in 1836, STURM, JOSEPH LIOUVILLE, JEAN-MARIE DUHAMEL, GABRIEL LAMÉ and JEAN-LOUIS BOUCHARLAT were suggested as successor candidates for the *Académie*. LIOUVILLE and DUHAMEL stood down because they considered STURM more suitable than themselves and he was accepted as a new member with a large majority. In 1837 STURM was admitted to the Legion of Honor (*Chévalier de l'ordre national de la Légion d'honneur*).

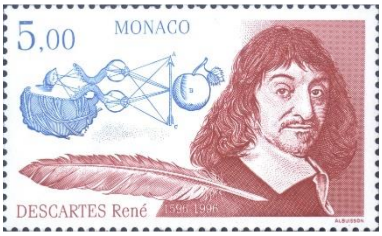
Together with LIOUVILLE, he developed a solution method for special second-order differential equations, $\frac{d}{dx}[p(x)\cdot\frac{dy}{dx}]+q(x)\cdot y=-\lambda\cdot w(x)\cdot y$ which play a special role in thermodynamics (STURM-LIOUVILLE theory).

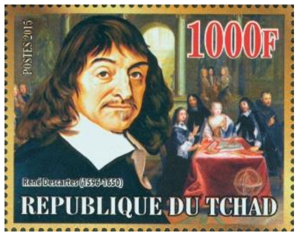
In 1838 STURM became *répétiteur* (assistant professor) of analysis, in 1840 he received the chair of analysis and mechanics at the *École Polytechnique* and in the same year he succeeded POISSON at the Sorbonne.

For ten years STURM gave his carefully prepared lectures on analysis and theoretical mechanics. After his death they appeared in printed form and for a long time were considered exemplary (there were 10 editions of each).

At the beginning of the 1850s, his health deteriorated so dramatically that the teacher, who was greatly admired by his students, gradually had to give up his duties. His burial in Montparnasse Cemetery was attended by representatives from science and politics.

When CHARLES FRANÇOIS STURM presented his method for determining the number of zeros of a polynomial in 1829, CHARLES HERMITE expressed the assumption that this elegant and in principle simple method would play a major role in the teaching of mathematics in the future. This, however, turned out to be wrong: STURM chains were never taught at school or at university.





In 1637, RENÉ DESCARTES formulated a simple sign rule in *La Géométrie*:

- *The number of positive zeros of an integral function f is the same as the number of sign changes in the sequence of the coefficients of f(x) or an even number less. The number of negative zeros can be calculated from the terms for f(-x).*

In the following, two examples will be used to explain which statements can be made using DESCARTES' and STURM's rules.

- Example 1: *How many roots does the function f with* $f(x)=x^3-x^2-4x+5$ *have*?

The coefficient sequence of *f*(*x*) is +1, -1, -4, +5; it has two sign changes. According to DESCARTES' criterion, the graph has *two* or *no* positive zeros.

The sequence of coefficients -1, -1, +4, +5 results for $f(-x)=-x^3-x^2+4x+5$, i.e. it has a change of sign. The graph therefore has *one* zero in the negative range.

Overall, the graph has *one* or *three* zeros.

Example of a STURM chain: $p_0(x)=f(x)=x^3-x^2-4x+5$, $p_1(x)=f'(x)=3x^2-2x-4$.

Next: $x^3-x^2-4x+5=(\frac{1}{3}x-\frac{1}{9})\cdot(3x^2-2x-4)-\frac{1}{9}\cdot(26x-41)$, since the prefactor $\frac{1}{9}$ is irrelevant, we have $\overline{p}_2(x)=26x-41$.

Finally: $3x^2-2x-4=(\frac{3}{26}x+\frac{71}{676})\cdot(26x-41)-(-\frac{207}{676})$, so $\overline{p}_3(x)=-\frac{207}{676}<0$.

This results in the sign of the STURM polynomials:

x	-3	-2	-1	0	+1	+2	+3
$p_0(x)$	< 0	> 0	> 0	> 0	> 0	> 0	> 0
$p_1(x)$	> 0	> 0	> 0	< 0	< 0	> 0	> 0
$p_2(x)$	< 0	< 0	< 0	< 0	< 0	> 0	> 0
$p_3(x)$	< 0	< 0	< 0	< 0	< 0	< 0	< 0
σ(*x*)	2	1	1	1	1	1	1

Hence it follows: The graph of *f* has in the interval

] -3 , -2] exactly σ(-3) – σ(-2) = 2 – 1 = 1 zero,

] -2 , -1] exactly σ(-2) – σ(-1) = 1 – 1 = 0 zeros,

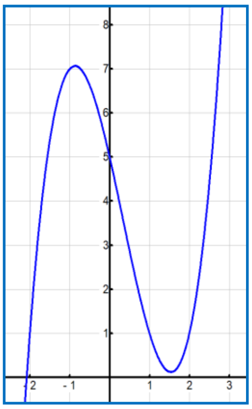
] -1 , 0] exactly σ(-1) – σ(0) = 1 – 1 = 0 zeros,

] 0 , +1] exactly σ(0) – σ(+1) = 1 – 1 = 0 zeros,

] +1 , +2] exactly σ(+1) – σ(+2) = 1 – 1 = 0 zeros,

] +2 , +3] exactly σ(+2) – σ(+3) = 1 – 1 = 0 zeros,

a total of 1 zero in the interval] -3 , +3].



- Example 2: *How many roots does the function f with* $f(x)=x^4-x^3-5x^2+2x+6$ *have*?

The coefficient sequence of *f*(*x*) is +1, -1, -5, +2, +6; it has two sign changes. According to DESCARTES' criterion, the graph has *two* or *no* positive zeros. For $f(-x)=x^4+x^3-5x^2-2x+6$ the sequence of coefficients is +1, +1, -5, -2, +6; it has *two* sign changes. The graph therefore has *two* or *no* zeros in the negative area.

Overall, the graph has *none*, *two* or *four* zeros.

STURM chain: $p_0(x)=f(x)=x^4-x^3-5x^2+2x+6$, $p_1(x)=f'(x)=4x^3-3x^2-10x+2$.

Next: $x^4-x^3-5x^2+2x+6=(4x^3-3x^2-10x+2)\cdot(\frac{1}{4}x-\frac{1}{16})-\frac{1}{16}\cdot(43x^2-14x-98)$, so $\overline{p}_2(x)=43x^2-14x-98$.

And: $4x^3-3x^2-10x+2=(43x^2-14x-98)\cdot(\frac{4}{43}x-\frac{73}{1849})-\frac{32}{1849}\cdot(83x+108)$, so:

$\overline{p}_2(x)=43x^2-14x-98$.

Finally: $43x^2-14x-98=(83x+108)\cdot(\frac{43}{83}x-\frac{5806}{6889})-\frac{48074}{6889}$, so: $\overline{p}_4(x)=1$.

This results in the sign of the STURM polynomials:

x	-3	-2	-1	0	+1	+2	+3
$p_0(x)$	> 0	> 0	> 0	> 0	> 0	< 0	> 0
$p_1(x)$	< 0	< 0	> 0	> 0	< 0	> 0	> 0
$p_2(x)$	> 0	> 0	< 0	< 0	< 0	> 0	> 0
$p_3(x)$	< 0	< 0	> 0	> 0	> 0	> 0	> 0
$p_4(x)$	> 0	> 0	> 0	> 0	> 0	> 0	> 0
$\sigma(x)$	4	4	2	2	2	1	0

Hence it follows: The graph of *f* has in the interval

] -3 , -2] exactly σ(-3) – σ(-2) = 4 – 4 = 0 zeros,

] -2 , -1] exactly σ(-2) – σ(-1) = 4 – 2 = 2 zeros,

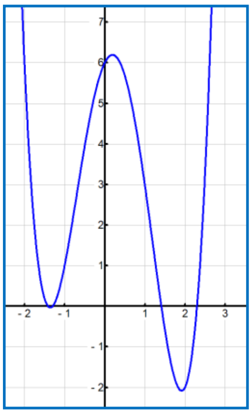
] -1 , 0] exactly σ(-1) – σ(0) = 2 – 2 = 0 zeros,

] 0 , +1] exactly σ(0) – σ(+1) = 2 – 2 = 0 zeros,

] +1 , +2] exactly σ(+1) – σ(+2) = 2 – 1 = 1 zero,

] +2 , +3] exactly σ(+2) – σ(+3) = 1 – 0 = 1 zero,

a total of 4 zeros in the interval] -3 , +3].



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