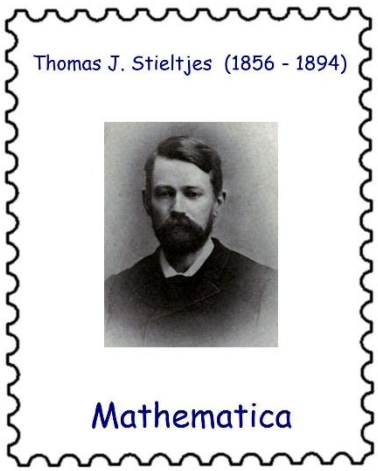
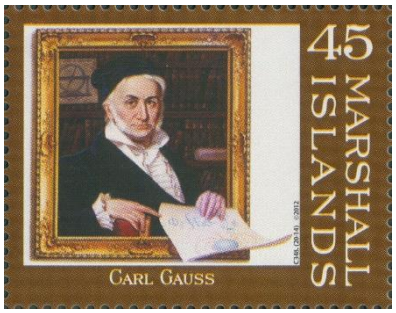
THOMAS STIELTJES (December 29, 1856 – December 31, 1894)

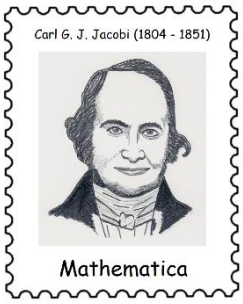
by HEINZ KLAUS STRICK, Germany



If you look for the mathematician THOMAS JOANNES STIELTJES in the encyclopaedia of famous Dutch personalities, you will surprisingly find references to a hydraulic engineer instead. In the middle of the 19th century, he created the conditions for Rotterdam to be one of the largest ports in Europe today by building the port in Feijenoord. He was also a member of parliament for many years as a Liberal MP. He had seven children – and to one of his three sons he gave exactly the first names he himself had.

The younger THOMAS JOANNES STIELTJES began studying at the Polytechnic School in Delft in 1873, but – instead of attending lectures – spent most of his time in the library, immersing himself in particular in the works of CARL FRIEDRICH GAUSS and CARL GUSTAV JACOB JACOBI.





(drawings © Andreas Strick)

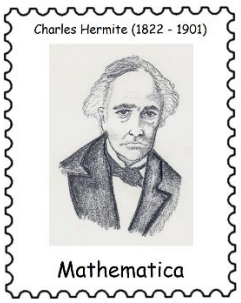
In 1876, his first paper was published. It dealt with the representation of functions with the help of other functions. And despite this unusual achievement for a student, he failed a compulsory examination – because he was not sufficiently familiar with the contents of the lecture in question. When he then failed this exam twice more in the following years, his father realised the seriousness of the situation and looked for a way out.

He made use of his many contacts: HENDRICUS GERARDUS VAN DE SANDE BAKHUYZEN, director of the observatory in Leiden, arranged a position for his friend's son as "assistant for astronomical calculations". This enabled the young man to earn his own living for the time being – and he had to, because his father died the following year.

After two papers in 1878 and 1880 (on properties of the gamma function and on a special sequence of mean values), STIELTJES published a total of twelve papers in various European journals in 1882 – on different topics from differential and integral calculus and on number theory.

In the course of his life, GAUSS had developed three different proofs of the fundamental theorem of algebra (*Every non-constant polynomial of the nth degree has exactly n zeros in the set of complex numbers*). STIELTJES presented another variant of the proof.

On one of the topics, he contacted CHARLES HERMITE, who was a professor of analysis at the *École Polytechnique* and the *Sorbonne* in Paris. This contact led to a lively correspondence over the next twelve years (until STIELTJES' premature death) – in total, this correspondence comprised 432 letters. In the years that followed, HERMITE repeatedly forwarded passages of text from STIELTJES' letters to the journal of the *Académie des Sciences* for publication immediately after receiving them.



1883 became an important year in STIELTJES' life. Encouraged also by his fiancée ELIZABETH INTVELD, whom he married in May, he asked VAN DE SANDE BAKHUYZEN to release him from astronomical observational tasks so that he could devote himself entirely to mathematical questions and his father's friend granted him this request.

At the end of the year STIELTJES took over lectures on Analytical and Descriptive Geometry at the University in Delft as a substitute for a lecturer who was ill.

Through positive feedback on his teaching, STIELTJES' self-confidence grew and he dared to take the next steps: quitting his job at the Observatory and applying for a vacant position as Professor of Analysis at the University of Groningen. In the course of the appointment procedure, although he was proposed by the university, the responsible ministry did not appoint him, but the second-placed candidate to the advertised position, as STIELTJES had no academic qualifications whatsoever.

In May 1884, HERMITE took part in the celebrations of the 300th anniversary of the University of Edinburgh, taking the opportunity to point out STIELTJES' unfortunate situation in conversation. BIERENS DE HAAN, an influential professor of mathematics at Leiden University, together with VAN DE SANDE BAKHUYZEN, then proposed to the Senate of his university that STIELTJES be awarded an honorary doctorate for his services in mathematics and astronomy. The Senate agreed to the proposal and STIELTJES was to be ceremoniously awarded the certificate at a public event.

Due to a misunderstanding, however, STIELTJES only learned of this date the day after the event ...

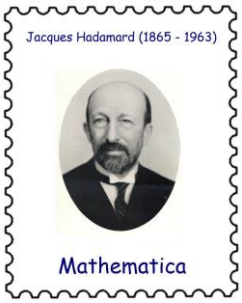
In 1885 STIELTJES moved to Paris with his young family. There he received his doctorate with a thesis on the convergence behaviour of special series at the *École Supérieure* (his supervisors were CHARLES HERMITE and GASTON DARBOUX).

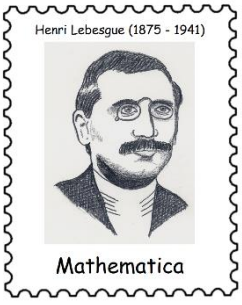
In the meantime, he had already published over 50 scientific papers.

To improve his chances of applying for a job in France, STIELTJES applied for French citizenship. In 1886 STIELTJES accepted a position at the University of Toulouse and from 1889 he was appointed to the chair of differential and integral calculus there. In the following years, too, numerous articles appeared in specialist journals and his achievements received international recognition. The *Koninklijke Nederlandse Akademie van Wetenschappen* accepted him as a member.

From 1890 onwards, STIELTJES was particularly concerned with complex-valued functions, which could be represented in the form of continued fractions:

$$f(z) = \cfrac{1}{a_1 z + \cfrac{1}{a_2 + \cfrac{1}{a_3 z + \cfrac{1}{a_4 + \ldots}}}}.$$





In 1892/93 he was awarded the *Prix Petit d'Ormoy* of the *Académie des Sciences* for his first publications on this subject (later prizewinners include JACQUES HADAMARD and HENRI LÉON LEBESGUE).

In 1894, the first 120-page part of his investigations was published and the second part appeared – posthumously – the following year.

In the context of this work, he introduced an integral that was later named after him. While in the RIEMANN integral one considers subdivisions of the integration interval $[a, b]$ by $a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$, the STIELTJES integral allows *almost any* subdivisions that can be described by a monotonically increasing function *g* (the *integrator*), i.e. instead of the RIEMANN sum $\sum_{i=1}^{n} f(\tau_i) \cdot (x_i - x_{i-1})$ with function values $f(\tau_i)$ for $x_{i-1} \le \tau_i \le x_i$ one examines the STIELTJES sum $\sum_{i=1}^{n} f(\tau_i) \cdot [\, g(t_i) - g(t_{i-1}) \,]$ for $t_{i-1} \le \tau_i \le t_i$. If the integrator function *g* is continuously differentiable, the method of partial integration is applicable and the following applies:

$$\int_a^b f(x)\,dg(x) = f(b)\,g(b) - f(a)\,g(a) - \int_a^b g(x)\,df(x)\,.$$

The importance of this generalisation of the integral concept by STIELTJES was only recognised at the beginning of the 20th century by the Hungarian mathematician FRIGYES RIESZ (1880-1956) and applied in functional analysis. A further generalisation of the integral concept was made in 1902 by HENRI LÉON LEBESGUE.

At the end of 1889, an influenza epidemic ("Asian flu") had broken out in Russia, which in the winter months of the following years repeatedly spread in waves over all European countries and, due to the intensive shipping connections, also found fatalities in North America – in total, more than one million people died.

In December 1894, THOMAS STIELTJES also fell victim to the epidemic. He died two days after his 38th birthday. Only a few days before his death, he had received the news that the *St Petersburg Russian Academy of Sciences* had appointed him a corresponding member.

First published 2019 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/thomas-stieltjes-ein-neuer-integralbegriff/1686320

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