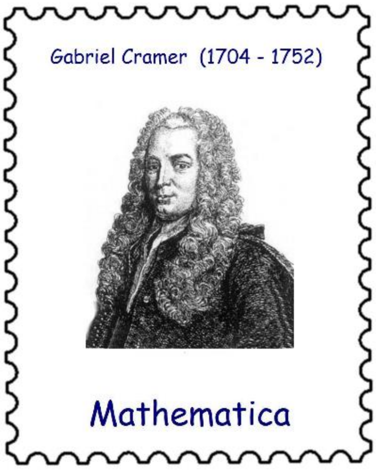
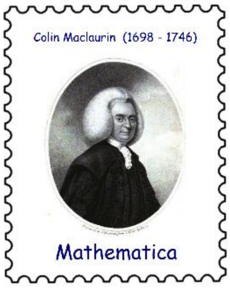
GABRIEL CRAMER (July 31, 1704 – January 4, 1752)

by HEINZ KLAUS STRICK, Germany



It was not so long ago that the name of the Swiss mathematician GABRIEL CRAMER was removed from the mathematics curricula of secondary schools in Germany. The CRAMER rule named after him (see below) offers a convenient way to solve simple linear systems of equations (with two or three equations or variables) without transformations.

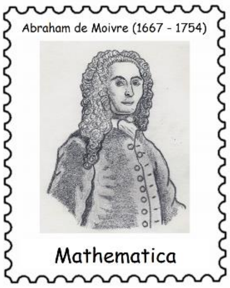
This rule had been discovered in 1678 by GOTTFRIED WILHELM LEIBNIZ, but not published. At about the same time as LEIBNIZ, SEKI KOWA developed a similar method in faraway Japan. However, it can hardly be assumed that his findings reached Europe very quickly. COLIN MACLAURIN had the same idea in principle as CRAMER (*Treatise of algebra*, 1748). However, he lacked a decisive step towards generalisation.

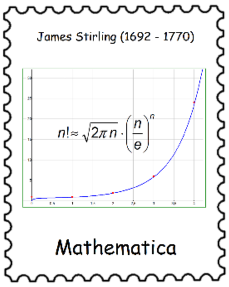


GABRIEL CRAMER enjoyed the best possible school education as the son of a respected Geneva doctor. At the age of 13 he transferred to the *Académie de Genève*, studied mathematics and philosophy under ÉTIENNE JALLABERT and he completed his studies at the age of 18 with a dissertation on the theory of sound. When JALLABERT died two years later, CRAMER applied to succeed him as professor of philosophy, together with his friend JEAN-LOUIS CALANDRINI, who was one year older. The magistrate of the city of Geneva decided in favour of a third applicant, the 26-year-old theologian AMÉDÉE DE LA RIVE. However, as the two young applicants had made an excellent impression, the city representatives made a wise decision: the philosophy chair was to be shared (this was also to counterbalance the influence of the church) and the additionally established professorship was to focus on mathematics. The two young applicants were commissioned to jointly take over the lectures on mathematics. CRAMER gave the lectures on geometry and mechanics, CALANDRINI those on algebra and astronomy.

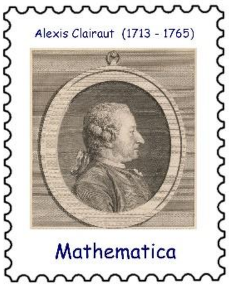
The magistrate's decision also contained another requirement: the young academics were obliged to inform themselves about the current state of research during a grand tour of European universities. During their absence, the other must assume full teaching duties at doubled salary.

In 1727, CRAMER began his two-year academic tour in Basel, where he worked with JOHANN and NICOLAUS BERNOULLI and met LEONHARD EULER before the latter accepted a call to St Petersburg. He then continued his journey and met EDMOND HALLEY, ABRAHAM DE MOIVRE and JAMES STIRLING, among others, in Cambridge and London.





He then went on to Paris via Leiden University (ALEXIS-CLAUDE CLAIRAUT, GEORGES-LOUIS LECLERC BUFFON, PIERRE LOUIS MOREAU DE MAUPERTUIS). He remained in correspondence with all these contemporaries until his death.

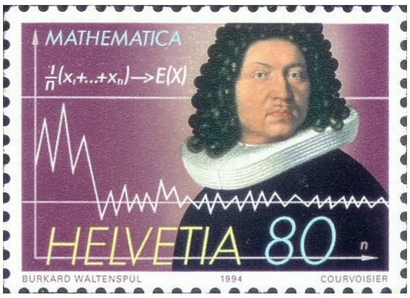


Returning to Geneva, he wrote a paper on the current prize question of the *Paris Académie Royale* on the cause of the elliptical shape of the planets and on the question of why the points furthest from the sun in the orbits of the planets are not fixed. Although JOHANN BERNOULLI won the prize, CRAMER's runner-up position helped his reputation grow in the academic world.

In 1734, his friend CALANDRINI took over the vacant chair of philosophy and from then on, CRAMER was solely responsible for the teaching of mathematics in Geneva.

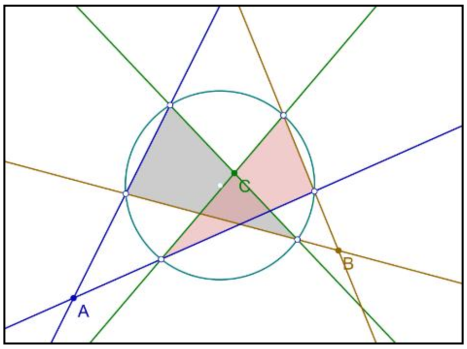
CRAMER's special scientific merits lies in two publications: on the one hand, his main work *Introduction à l'analyse des lignes courbes algébriques*, which appeared in 1750, and on the other hand, the publication of the *Collected Works* of JOHANN BERNOULLI.

The latter had expressed the wish that the large number of his own writings be sifted and compiled during his lifetime, and for him only GABRIEL CRAMER could be considered for this task. After the successful completion of the laborious and important work, in 1742, JOHANN BERNOULLI also handed over to CRAMER the writings of his brother JACOB BERNOULLI, who had died in 1705, and two years later these *Collected Works* also appeared – with the exception of the *Ars conjectandi*, which JACOB's nephew NICOLAUS BERNOULLI had already completed and published in 1713.



In collaboration with the mathematician Jean De Castillon (Giovanni Francesco Salvemini da Castiglione), who taught in Lausanne, he then published Johann Bernoulli's correspondence with Leibniz.

Castillon found a solution to the following geometrical problem twenty years after it was posed to him by Cramer. (In the meantime Castillon was the first astronomer of the Royal Observatory in Berlin.) (Diagram: Rocchini Wikipedia)



- Cramer-Castillon problem: Given a circle k and three points A, B, C which do not lie on the circle line, construct all possible triangles whose circumcircle is k and whose sides pass through the three points.

Cramer was inspired to write his 700-page main work on algebraic curves in particular by Newton's work on cubic curves and by Stirling's additions to it.

After introductory chapters in which he explained how curves can be drawn and how the equations of the curves could be simplified by suitable transformations, in the third chapter he developed a classification of algebraic curves by the degree of the defining polynomial.

Algebraic curves of degree 1 are defined by polynomials of the type $ax+by+c$, curves of degree 2 by $ax^2+bxy+cy^2+dx+ey+f$ etc. - for the corresponding curve equations ($ax+by+c=0$, $ax^2+bxy+cy^2+dx+ey+f=0$ etc.) three coefficients (2 for degree 1 and 1 for degree 0) or six coefficients (3 for degree 2; 2 for degree 1 and 1 for degree 0) are needed, thus $\frac{1}{2}\cdot(n+1)\cdot(n+2)$ coefficients are needed for a curve of degree n. Since the number can be reduced by 1 by dividing by any of the non-zero coefficients, it follows that $\frac{1}{2}\cdot(n+1)\cdot(n+2)-1=\frac{1}{2}\cdot n\cdot(n+3)$ coefficients are sufficient for the definition. One therefore has:

- Cramer's theorem:
  The necessary and sufficient number of points by which an algebraic curve of the nth degree lying in the plane is uniquely defined is $\frac{1}{2}\cdot n\cdot(n+3)$ – unless a degenerate case exists.

This is because inserting the $\frac{1}{2}\cdot n\cdot(n+3)$ pairs of point coordinates into the equation of the curve results in a linear system of equations with $\frac{1}{2}\cdot n\cdot(n+3)$ equations and just as many variables from which the coefficients can be unambiguously determined. However, the selection of points is not completely arbitrary (*degenerate* case): Three of the selected points must not lie on a straight line.

Cramer noticed a paradoxical situation in the case of cubic curves, which he could not explain himself: According to a theorem of Maclaurin, algebraic curves of the mth and nth degree can have up to $m \cdot n$ common points, i.e. for $m = n = 3$: two cubic curves can intersect at 9 points. If one now selects exactly these 9 intersections, then there should be only one cubic curve for this – but two curves exist for the selected points (Cramer's paradox).

In the appendix of the book, CRAMER finally gave the rule named after him; he described in words the procedure according to which the solution terms are formed for systems of equations up to the 5th order.

- CRAMER's rule for 2×2 systems of equations

The 2nd order linear system of equations $\left| \begin{array}{l} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{array} \right|$ has the solution

$x_1 = \dfrac{b_1 \cdot a_{22} - b_2 \cdot a_{12}}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$, $x_2 = \dfrac{a_{11} \cdot b_2 - a_{21} \cdot b_1}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$, if $a_{11} \cdot a_{22} - a_{21} \cdot a_{12} \neq 0$.

The busy CRAMER – in the meantime he also held municipal offices and took care of the city fortifications, for example – suffered a health breakdown at the end of 1751 after an accident with a carriage. On his way to the south of France, where he was supposed to recover from his exertions, he died before he reached his destination.

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https://www.spektrum.de/wissen/gabriel-cramer-ein-wahrer-networker/1613532

Translated 2022 by John O’Connor, University of St Andrews

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