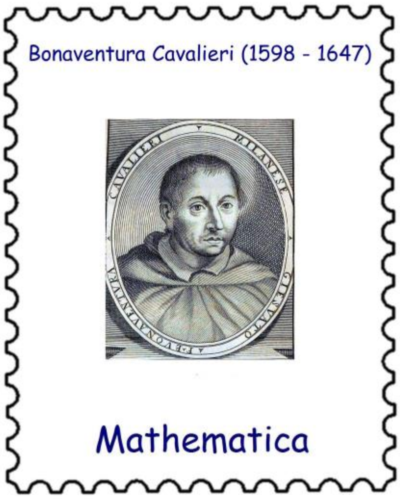
Bonaventura Cavalieri (1598 – November 30, 1647)

by Heinz Klaus Strick, Germany



At the age of 17, Francesco Cavalieri, a native of Milan, entered the Jesuit Order, adding the first name Bonaventura (this was his father's first name).

The lay organisation of the Jesuits had been founded in the 14th century to obtain salvation through special works of charity such as care of the plague sick and burial of the plague dead. By 1600, the number of friars in the order had declined sharply, and only a few young men responded to renewed calls to join the order – among them Francesco Cavalieri.

After stays in the monasteries of the Order in Milan and Florence, he came to the branch in Pisa. Here also lived the Benedictine monk Benedetto Antonio Castelli. The latter, a former pupil of Galileo Galilei, taught as a mathematics professor at the university and lived in the monastery of the Jesuits, as his order did not have its own monastery in Pisa. Castelli aroused Cavalieri's interest in mathematics, especially in the elements of Euclid.

Castelli recognised the extraordinary perceptiveness and high mathematical talent of his new pupil and arranged a first meeting between Cavalieri and Galileo, from which a lively correspondence and a lifelong friendship developed. Cavalieri later described himself as a pupil of Galileo, which he could substantiate with over one hundred correspondence contacts.

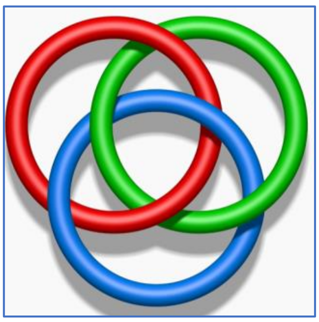
Cavalieri quickly attained such professional sovereignty that he was able to take over Castelli's lectures on a substitute basis. In 1619 he even applied for a vacant chair of mathematics in Bologna, but he was still considered too young for this post.

When other applications also failed, he attributed this to the fact that the Jesuit order was not very popular with the official church. Whether this really played a role can no longer be clarified, but obviously his membership of the order did not affect his ecclesiastical career.

In 1621 Cavalieri became deacon and assistant to Cardinal Federico Borromeo of Milan. From 1623 he was prior of the Jesuit monastery in Lodi (30 km south of Milan), and from 1626 to 1629 head of the monastery in Parma.

Borromean rings

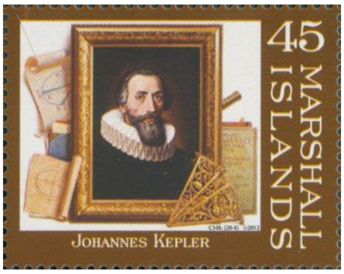
The coat of arms of the influential Italian noble family of the Borromeo contains, among other things, the so-called Borromean rings: if you open one of the three intertwined rings, the other two are also free.



(source: Wikimedia)

Already during his time in Milan CAVALIERI had occupied himself with the idea of the indivisibles and at the end of 1627 he informed GALILEO and Cardinal BORROMEO that he had completed the work on the book *Geometria indivisibilibus continuorum nova quadam ratione promota.*

With GALILEO's express recommendation (*... few, if any, have delved so deeply into the science of geometry since ARCHIMEDES ...*), he was appointed to the chair of mathematics at the University of Bologna in 1629, and shortly afterwards was also appointed prior of the monastery there, so that he had ideal working conditions for further research.



In his 1615 work *Nova Stereometria doliorum vinariorum*, JOHANNES KEPLER had shown how the volume of wine barrels could be determined with the help of an infinite number of thin discs. CAVALIERI took up this idea:

- *A surface can be thought of as divided into an infinite number of "indivisible" arcs parallel to one another, a body correspondingly into planes parallel to one another,*
  in general: a geometric structure can be divided into an infinite number of indivisibles of the next smallest dimension.

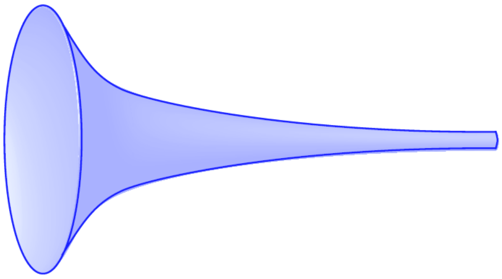
In the spatial case, the body under consideration is bounded by two mutually parallel planes; one plane then "flows" parallel in the direction of the other bounding plane and gives rise to an infinite number of intersecting surfaces, the totality of which then make up the body. In the theory of indivisibles, the infinitely many intersecting surfaces are regarded as the *leaves of a book*, which, however, since they are infinitely thin, cannot be divided further (*indivisibilis*, Latin = indivisible).

Correspondingly, in the plane case of a surface, the surface is bounded by two mutually parallel straight lines, between which lie the infinitely many infinitely thin straight line pieces that make up the surface like the *threads of a cloth*.

After the publication of the book in 1635, CAVALIERI's ideas were not only met with approval. The most severe critic was the Jesuit PAUL GULDIN (still known today because of the *GULDIN rules* named after him, with the help of which one can calculate the volume and surface of bodies of revolution). He accused CAVALIERI of plagiarism from KEPLER, but above all he pointed to irresolvable contradictions of CAVALIERI's theory. The latter was able to refute the accusations of plagiarism – obviously GULDIN did not see the differences to KEPLER's approach.

For the strict geometer GULDIN, however, especially in the consideration of the *infinitely many* objects which are *infinitely thin*, the exactness otherwise usual in mathematics was missing. CAVALIERI was convinced of the correctness of his method and this was shown by the fact that he has succeeded in deriving some previously unknown results with the help of the principles he had established.

GULDIN's reputation within his order was so great that other Jesuits felt compelled to fight the spread of CAVALIERI's theories in Italy. EVANGELISTA TORRICELLI was not impressed by these criticisms, but took up CAVALIERI's ideas and developed them further (including his investigation of the hyperbolic body of revolution with finite volume reaching into infinity: *Gabriel's Horn*).



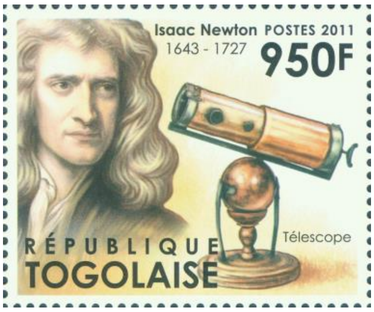
CAVALIERI himself published a paper in 1647 in which he demonstrated the correctness of his method in detail by means of six examples (*Exercitationes geometricae sex*).

CAVALIERI's appointment to the chair in Bologna was initially only for a period of three years.

However, he consolidated his position by publishing several of GALILEO writings. He was the first mathematician in Italy to propagate calculating with logarithms and he himself published a table of logarithms, including for the values of trigonometric functions.

In addition, he went into NAPIER's rules for spherical triangles in depth. In another book, he explained logarithmic arithmetic by means of one hundred examples, of which GALILEO was very impressed.

He also dealt theoretically with the construction of telescopes and considered how mirrors and lenses could be combined (which was later constructed by ISAAC NEWTON).



Around 1629 CAVALIERI had his first attacks of gout and in the course of the years his work was increasingly affected by it. A stay at a health resort in Arcetri did not bring any relief (but enabled him to have many fruitful conversations with GALILEO, who had been living there under house arrest since his trial). From 1646 onwards he was no longer able to leave his home and the following year he died.

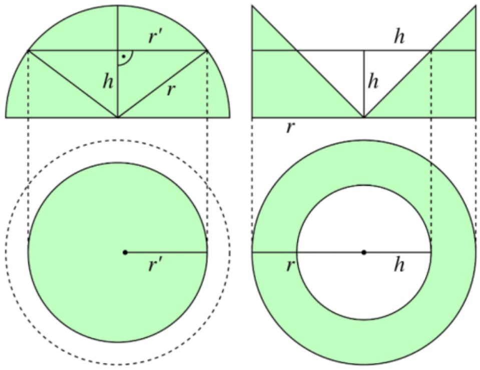
The idea of the indivisible represented an important step in the development of the integral calculus. With the help of his method, CAVALIERI was able to derive, for example, the relation

$\int_0^1 x^n dx = \frac{1}{n+1}$, $n \in \mathbb{N}$ (written down in today's notation).

CAVALIERI's principle is still considered a standard method of plane and spatial geometry today.

- *If two plane figures are intersected by a set of parallel straight lines in such a way that each of these straight lines cuts out sections of equal length in both figures, then the two figures are equal in area.*
- *If two solids are intersected by a set of parallel planes in such a way that each of these planes cuts out equal areas in both solids, then the two solids are equal in volume.*

As a prime example, we may refer to the determination of the volume of the hemisphere as the difference between the cylinder and cone volume, cf. fig. (from Wikipedia)



The following also applies:

- *If the intersections (length of line or size of area) of the plane or spatial figures under consideration are always in the same ratio, then the areas or volumes of the figures under consideration are also in this ratio.*

First published 2018 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/bonaventura-cavalieri-vom-krankenpfleger-zum-mathematiker/1565832

Translated 2021 by John O'Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps.

Enquiries at europablocks@web.de with the note: "Mathstamps".